Extra Practice with Normal Distributions I

There are two major tests of readiness for college, the ACT and the SAT. ACT scores are reported on a scale from 1 to 36. The distribution of ACT scores in recent years has been roughly Normal with mean μ = 20.9 *and standard deviation* σ = 4.8. *SAT scores are reported on a scale from 600 to 2400. The distribution of SAT scores in 2013 was roughly Normal with mean* μ = 1499 *and standard deviation* σ = 319. *The following exercises are based on this information.*

1. Jose scores 1770 on the SAT. Assuming that both tests measure the same thing, what score on the ACT is equivalent to Jose's SAT score? Explain.

2. Reports on a student's ACT or SAT usually give the percentile as well as the actual score. Tonya scores 1320 on the SAT. What is her percentile? Show your method.

3. The quartiles of any distribution are the values with cumulative proportions 0.25 and 0.75. What are the quartiles of the distribution of ACT scores? Show your method.

Extra Practice with Normal Distributions II

There are two major tests of readiness for college, the ACT and the SAT. ACT scores are reported on a scale from 1 to 36. The distribution of ACT scores in recent years has been roughly Normal with mean μ = 20.9 *and standard deviation* σ = 4.8. *SAT scores are reported on a scale from 600 to 2400. The distribution of SAT scores in 2013 was roughly Normal with mean* μ = 1499 *and standard deviation* σ = 319. *The following exercises are based on this information.*

1. Maria scores 28 on the ACT. Assuming that both tests measure the same thing, what score on the SAT is equivalent to Maria's ACT score? Explain.

2. Reports on a student's ACT or SAT usually give the percentile as well as the actual score. Jacob scores 16 on the ACT. What is his percentile? Show your method.

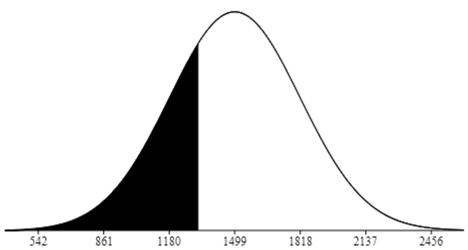
3. The quintiles of any distribution are the values with cumulative proportions 0.20, 0.40, 0.60, and 0.80. What are the quintiles of the distribution of SAT scores? Show your method.

Answers to Extra Practice with Normal Distributions

Extra Practice I:

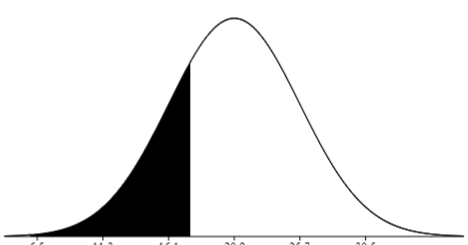
1. On the SAT, Jose's *z*-score is $z = \dfrac{1770 - 1499}{319} = 0.85$. To find his equivalent score on the ACT, we solve $0.85 = \dfrac{x - 20.9}{4.8}$ for *x* and get *x* = 24.98.

2. Step 1: State the distribution and values of interest. For the SAT, scores follow a Normal distribution with mean 1499 and standard deviation 319. We want to find the percent of students with scores less than 1320 (see graph below). **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \dfrac{1320 - 1499}{319} = -0.56$. From Table A, the proportion of *z*-scores below –0.56 is 0.2877. *Using technology:* The command normalcdf(lower: –1000, upper: 1320, μ: 1499, σ: 319) gives an area of 0.2874. **Step 3: Answer the question.** Tanya's score is at the 29th percentile.

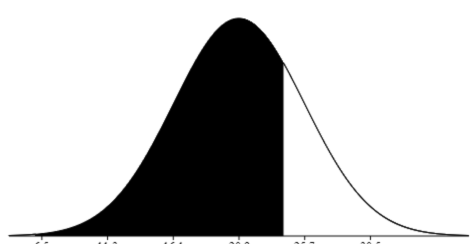


SAT scores

3. Step 1: State the distribution and values of interest. For the ACT, scores follow a Normal distribution with mean 20.9 and standard deviation 4.8. The 25^{h} percentile is the boundary value *x* with 25% of the distribution to its left (see graph below). Likewise, the 75^{h} percentile is the boundary value *x* with 75% of the distribution to its left (see graph below). **Step 2: Perform calculations. Show your work.** Look in the body of Table A for a value closest to 0.25. A *z*-score of –0.67 gives the closest value (0.2514). Solving $-0.67 = \dfrac{x - 20.9}{4.8}$ gives *x* = 17.7. *Using technology:* The command invNorm(area: 0.25, μ: 20.9, σ: 4.8) gives a value of 17.7. Likewise, solving $0.67 = \dfrac{x - 20.9}{4.8}$ gives *x* = 24.1. **Step 3: Answer the question.** For the ACT, the quartiles are 17.7 and 24.1.



ACT scores

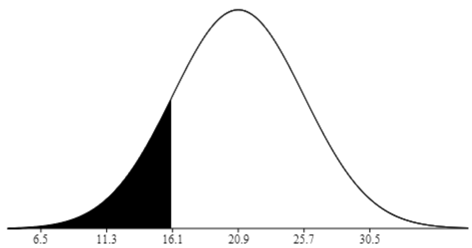


ACT scores

Extra Practice II:

1. On the ACT, Maria's *z*-score is $z = \dfrac{28 - 20.9}{4.8}$ = 1.48. To find her equivalent score on the SAT, we solve $1.48 = \dfrac{x - 1499}{319}$ for *x* and get *x* = 1971.

2. Step 1: State the distribution and values of interest. For the ACT, scores follow a Normal distribution with mean 20.9 and standard deviation 4.8. We want to find the percent of students with scores less than 16 (see graph below). **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \dfrac{16 - 20.9}{4.8} = -1.02$. From Table A, the proportion of *z*-scores below –1.02 is 0.1539. *Using technology:* The command normalcdf(lower: –1000, upper:16 , μ: 20.9, σ: 4.8) gives an area of 0.1537. **Step 3: Answer the question.** Jacob's score is at the 15th percentile.



ACT scores

3. Step 1: State the distribution and values of interest. For the SAT, scores follow a Normal distribution with mean 1499 and standard deviation 319. The 20^{h} percentile is the boundary value *x* with 20% of the distribution to its left (see graph below). **Step 2: Perform calculations. Show your work.** Look in the body of Table A for a value closest to 0.20. A *z*-score of –0.84 gives the closest value (0.2005). Solving $-0.84 = \dfrac{x - 1499}{319}$ gives *x* = 1231. *Using technology:* The command invNorm(area: 0.20, μ: 1499, σ: 319) gives a value of 1231. Likewise, solving $-0.25 = \dfrac{x - 20.9}{4.8}$ gives *x* = 1419, solving $0.25 = \dfrac{x - 20.9}{4.8}$ gives *x* = 1579, and solving $0.84 = \dfrac{x - 20.9}{4.8}$ gives *x* = 1767. **Step 3: Answer the question.** For the SAT, the quintiles are 1231, 1419, 1579, and 1767.

SAT scores