AP STATISTICS EXAM: 3 HOURS

Exam questions are based on the topics and skills addressed in the AP Statistics course. Formulas and tables needed to complete exam questions are provided to students taking the exam. Students are allowed to use a graphing calculator with statistical capabilities on the entire exam.

Format of Assessment

Section I: Multiple Choice | 40 Questions | 90 Minutes | 50% of Exam Score

- Discrete Questions

Section II: Free Response | 6 Questions | 90 Minutes | 50% of Exam Score

- 5 Short-Answer Questions
- 1 Investigative Task

1997

68-100 5
54-67 4
41-53 3
29-40 2
0-28 1

2002

68-100 5
53-67 4
40-52 3
29-39 2
0-28 1

2007

60-100 5
45-59 4
32-44 3
23-31 2
0-22 1

Introduction

Suppose we want to compare the proportions of individuals with a certain characteristic in Population 1 and Population 2. Let's call these parameters of interest p_1 and p_2. The ideal strategy is to take a separate random sample from each population and to compare the sample proportions with that characteristic.

What if we want to compare the effectiveness of Treatment 1 and Treatment 2 in a completely randomized experiment? This time, the parameters p_1 and p_2 that we want to compare are the true proportions of successful outcomes for each treatment. We use the proportions of successes in the two treatment groups to make the comparison.

Population or treatment	Parameter	Statistic	Sample size
1	p_1	$\hat{p}_1$	n_1
2	p_2	$\hat{p}_2$	n_2

The Sampling Distribution of a Difference Between Two Proportions

To explore the sampling distribution of the difference between two proportions, let' s start with two populations having a known proportion of successes.

- ✓ At School 1, 70% of students did their homework last night
- ✓ At School 2, 50% of students did their homework last night.

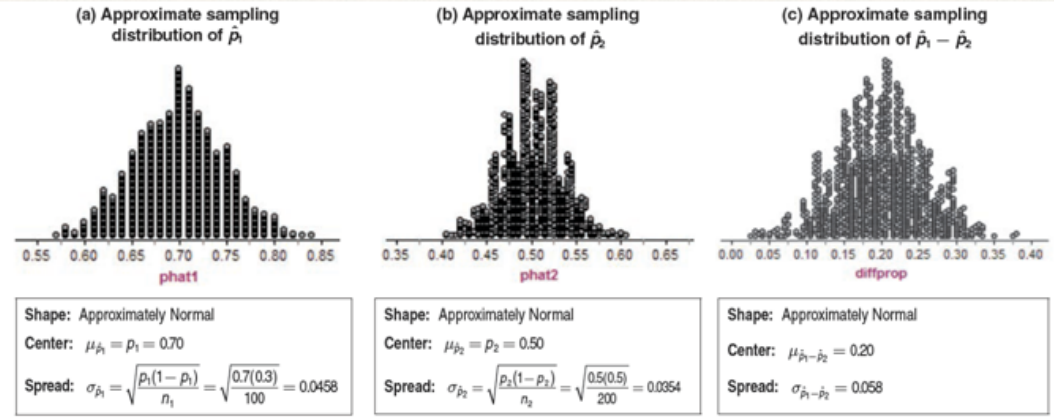
Suppose the counselor at School 1 takes an SRS of 100 students and records the sample proportion that did their homework.

School 2's counselor takes an SRS of 200 students and records the sample proportion that did their homework.

What can we say about the difference $\hat{p}_1 - \hat{p}_2$ in the sample proportions?

The Sampling Distribution of a Difference Between Two Proportions

Using Fathom software, we generated an SRS of 100 students from School 1 and a separate SRS of 200 students from School 2. The difference in sample proportions was then be calculated and plotted. We repeated this process 1000 times.



What do you notice about the shape, center, and spread of the sampling distribution of $\hat{p}_1 - \hat{p}_2$?

The Sampling Distribution of a Difference Between Two Proportions

Both $\hat{p}_1$ and $\hat{p}_2$ are random variables. The statistic $\hat{p}_1 - \hat{p}_2$ is the difference of these two random variables. In Chapter 6, we learned that for any two independent random variables X and Y,

$\mu_{X-Y} = \mu_X - \mu_Y$ and $\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$

The Sampling Distribution of the Difference Between Sample Proportions

Choose an SRS of size n_1 from Population 1 with proportion of successes p_1 and an independent SRS of size n_2 from Population 2 with proportion of successes p_2.

Shape When n_1p_1, $n_1(1-p_1)$, n_2p_2 and $n_2(1-p_2)$ are all at least 10, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal.

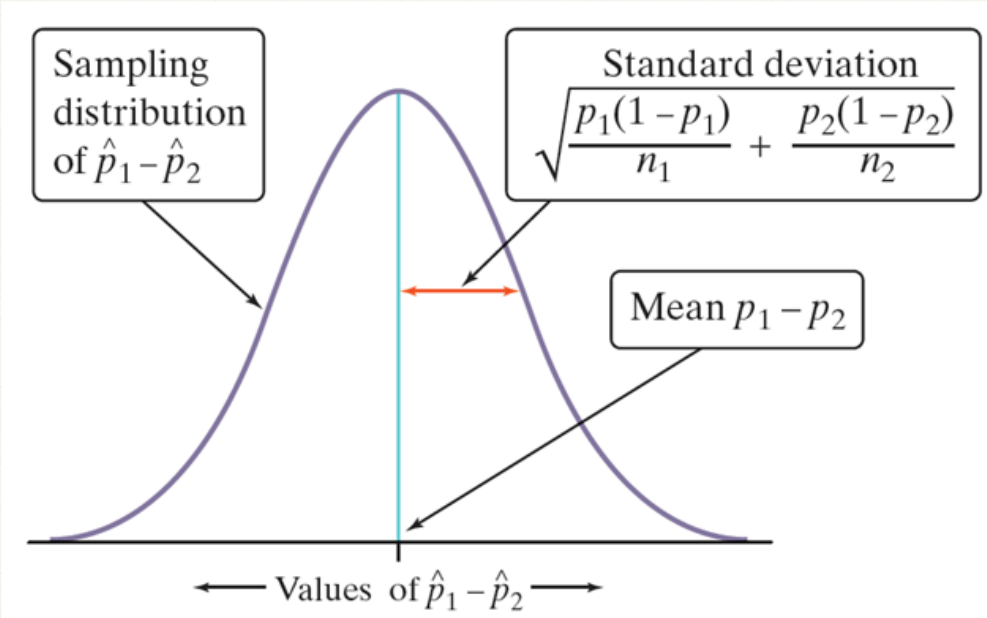
Center The mean of the sampling distribution is $p_1 - p_2$.

Spread The standard deviation of the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is

$$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

as long as each sample is no more than 10% of its population (10% condition)

The Sampling Distribution of a Difference Between Two Proportions



The Sampling Distribution of a Difference Between Two Proportions

Suppose that there are two large high schools, each with more than 2000 students, in a certain town. At School 1, 70% of students did their homework last night. Only 50% of the students at School 2 did their homework last night. The counselor at School 1 takes an SRS of 100 students and records the proportion that did homework. School 2's counselor takes an SRS of 200 students and records the proportion that did homework

a) Describe the shape, center, and spread of the sampling distribution of $\hat{p}_1 - \hat{p}_2$.

Because $n_1p_1 = 100(0.7) = 70$, $n_1(1-p_1) = 100(0.30) = 30$, $n_2p_2 = 200(0.5) = 100$ and $n_2(1-p_2) = 200(0.5) = 100$ are all at least 10, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal.

Its mean is $p_1 - p_2 = 0.70 - 0.50 = 0.20$.

Its standard deviation is $\sqrt{\frac{0.7(0.3)}{100} + \frac{0.5(0.5)}{200}} = 0.058$.

American made cars

Nathan and Kyle both work for the Department of Motor Vehicles (DMV), but they live in different states. In Nathan's state, 80% of the registered cars are made by American manufacturers. In Kyle's state, only 60% of the registered cars are made by American manufacturers. Nathan selects a random sample of 100 cars in his state and Kyle selects a random sample of 70 cars in his state. Let $\hat{p}_N - \hat{p}_K$ be the difference in the sample proportion of cars made by American manufacturers.

Problem:

(a) What is the shape of the sampling distribution of $\hat{p}_N - \hat{p}_K$? Why?

(b) Find the mean of the sampling distribution. Show your work.

(c) Find the standard deviation of the sampling distribution. Show your work.

Confidence Intervals for $p_1 - p_2$

When data come from two random samples or two groups in a randomized experiment, the statistic $\hat{p}_1 - \hat{p}_2$ is our best guess for the value of $p_1 - p_2$. We can use our familiar formula to calculate a confidence interval for $p_1 - p_2$:

$$\text{statistic} \pm (\text{critical value}) \cdot (\text{standard deviation of statistic})$$

When the 10% condition is met, the standard deviation of the statistic $\hat{p}_1 - \hat{p}_2$ is:

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

If the Normal condition is met, we find the critical value z^* for the given confidence level from the standard Normal curve.

Confidence Intervals for $p_1 - p_2$

Conditions For Constructing A Confidence Interval About A Difference In Proportions

- **Random**: The data come from two independent random samples or from two groups in a randomized experiment.
  - **10%**: When sampling without replacement, check that $n_1 \le (1/10)N_1$ and $n_2 \le (1/10)N_2$.
- **Large Counts**: The counts of "successes" and "failures" in each sample or group -- $n_1\hat{p}_1$, $n_1(1-\hat{p}_1)$, $n_2\hat{p}_2$ and $n_2(1-\hat{p}_2)$ -- are all at least 10.

Because we don't know the values of the parameters p_1 and p_2, we replace them in the standard deviation formula with the sample proportions. The result is the **standard error** of the statistic $\hat{p}_1 - \hat{p}_2$:

$$\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Confidence Intervals for $p_1 - p_2$

Two-Sample z Interval for a Difference Between Two Proportions

When the conditions are met, an approximate C% confidence interval for $\hat{p}_1 - \hat{p}_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

where z^* is the critical value for the standard Normal curve with C% of its area between $-z^*$ and z^*.

The researchers in the previous example selected independent random samples from the two populations they wanted to compare. In practice, it's common to take one random sample that includes individuals from both populations of interest and then to separate the chosen individuals into two groups. The two-sample z procedures for comparing proportions are still valid in such situations, provided that the two groups can be viewed as independent samples from their respective populations of interest.

You can use technology to perform the calculations in the "Do" step. Remember that this comes with potential benefits and risks on the AP® exam.

21. TECHNOLOGY CORNER CONFIDENCE INTERVAL FOR A DIFFERENCE IN PROPORTIONS

The TI-83/84 and TI-89 can be used to construct a confidence interval for $p_1 - p_2$. We'll demonstrate using the previous example. Of $n_1 = 799$ teens surveyed, $X = 639$ said they used social-networking sites. Of $n_2 = 2253$ adults surveyed, $X = 1555$ said they engaged in social networking. To construct a confidence interval:

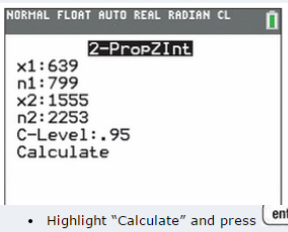
TI-83/84

- Press STAT, then choose TESTS and 2-PropZInt.

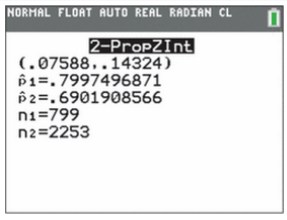
TI-89

- In the Statistics/List Editor, press 2nd F2 ([F7]) and choose 2-PropZInt.

- When the 2-PropZInt screen appears, enter the values shown.



- Highlight "Calculate" and press enter.



Presidential approval

Many news organizations conduct polls asking adults in the United States if they approve of the job the president is doing. How did President Obama's approval rating change from October 2012 to October 2013? According to a Gallup poll of 1500 randomly selected U.S. adults on October 2–4, 2012, 52% approved of Obama's job performance. A Gallup poll of 1500 randomly selected U.S. adults on October 5–7, 2013, showed that 46% approved of Obama's job performance.

Problem:

(a) Calculate the standard error of the sampling distribution of the difference in the sample proportions (2013 – 2012). Interpret this value.

(b) Use the results of these polls to construct and interpret a 90% confidence interval for the change in Obama's approval rating among all U.S. adults from October 2012 to October 2013.

(c) Based on your interval, is there convincing evidence that Obama's job approval rating has changed?

a) $SE = \sqrt{\frac{.46(1-.46)}{1500} + \frac{.52(1-.52)}{1500}} = .0182$

$x_1 = 780$
$n_1 = 1500$
$x_2 = 690$
$n_2 = 1500$

b) Check Assumptions 2prop Z Int

$\hat{P}_{12} - \hat{P}_{13}$ (.03, .09)

$\hat{P}_{13} - \hat{P}_{12}$ (−.09, −.03)

CHECK YOUR UNDERSTANDING

AP® EXAM TIP The formula for the two-sample z interval for $p_1 - p_2$ often leads to calculation errors by students. As a result, we recommend using the calculator's 2-PropZInt feature to compute the confidence interval on the AP® exam. Be sure to name the procedure (two-proportion z interval) and to give the interval (0.076, 0.143) as part of the "Do" step.

Are teens or adults more likely to go online daily? The Pew Internet and American Life Project asked a random sample of 799 teens and a separate random sample of 2253 adults how often they use the Internet. In these two surveys, 63% of teens and 68% of adults said that they go online every day. Construct and interpret a 90% confidence interval for $p_1 - p_2$.

State: p_1 = true prop. of teens who go online everyday

p_2 = true prop. of adults who go online everyday

Plan: 2 prop. z int for $p_1 - p_2$

random ✓ Indep. random samples

10% ✓ more than 7990 teens & 22530 adults in US

Large Counts ✓

.63(799) = 503 799 − 503 = 296 ≥ 10

.68(2253) = 1532 2253 − 1532 = 721 ≥ 10

Do: (−.083, −.018)

Conclude: We are 90% confident that the interval −.083 to −.018 captures true difference in prop. between teens & adults who go online each day

Significance Tests for $p_1 - p_2$

An observed difference between two sample proportions can reflect an actual difference in the parameters, or it may just be due to chance variation in random sampling or random assignment. Significance tests help us decide which explanation makes more sense.

The null hypothesis has the general form

$$H_0: p_1 - p_2 = \text{hypothesized value}$$

We'll restrict ourselves to situations in which the hypothesized difference is 0. Then the null hypothesis says that there is no difference between the two parameters:

$$H_0: p_1 - p_2 = 0 \text{ or, alternatively, } H_0: p_1 = p_2$$

The alternative hypothesis says what kind of difference we expect.

$$H_a: p_1 - p_2 > 0,\ H_a: p_1 - p_2 < 0, \text{ or } H_a: p_1 - p_2 \neq 0$$

Hearing loss

Are teenagers going deaf? In a study of 3000 randomly selected teenagers in 1988–1994, 15% showed some hearing loss. In a similar study of 1800 teenagers in 2005–2006, 19.5% showed some hearing loss. Do these data give convincing evidence that the proportion of all teens with hearing loss has increased? (These data were reported in *Arizona Daily Star,* August 18, 2010.)

Problem: State the hypotheses we are interested in testing. Define any parameters you use.

P_1 = prop. of teens w/ hearing loss 88-94

P_2 = prop. of teens w/ hearing loss 05-06

$H_0: P_1 - P_2 = 0$ $\quad P_1 = P_2$

$P_1 - P_2 < 0$ $\quad H_a: P_1 < P_2$

Significance Tests for $p_1 - p_2$

Conditions For Performing a Significance Test About A Difference In Proportions

- **Random**: The data come from two independent random samples or from two groups in a randomized experiment.
  - **10%**: When sampling without replacement, check that $n_1 \leq (1/10)N_1$ and $n_2 \leq (1/10)N_2$.
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Significance Tests for $p_1 - p_2$

To do a test, standardize $\hat{p}_1 - \hat{p}_2$ to get a z statistic:

$$\text{test statistic} = \frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\text{standard deviation of statistic}}$$

If H_0: $p_1 = p_2$ is true, the two parameters are the same. We call their common value p. We need a way to estimate p, so it makes sense to combine the data from the two samples. This **pooled** (or **combined**) **sample proportion** is:

$$\hat{p}_C = \frac{\text{count of successes in both samples combined}}{\text{count of individuals in both samples combined}} = \frac{X_1 + X_2}{n_1 + n_2}$$

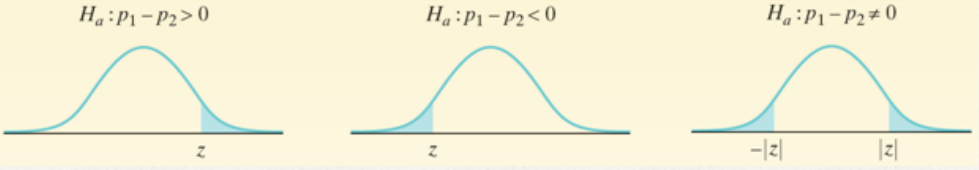
Significance Tests for $p_1 - p_2$

Two-Sample z Test for the Difference Between Two Proportions

Suppose the conditions are met. To test the hypothesis $H_0: p_1 - p_2 = 0$, first find the pooled proportion $\hat{p}_C$ of successes in both samples combined Then compute the z statistic

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\frac{\hat{p}_C(1-\hat{p}_C)}{n_1} + \frac{\hat{p}_C(1-\hat{p}_C)}{n_2}}}$$

Find the P-value by calculating the probabilty of getting a z statistic this large or larger in the direction specified by the alternative hypothesis H_a:



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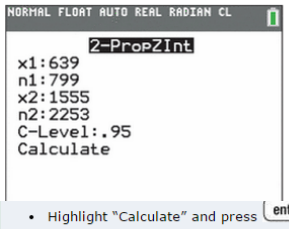

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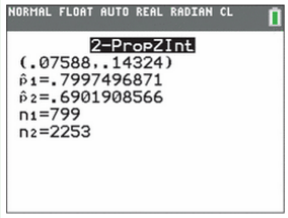
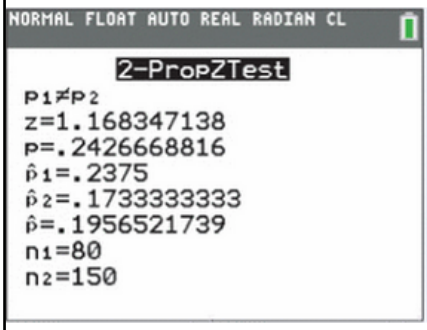
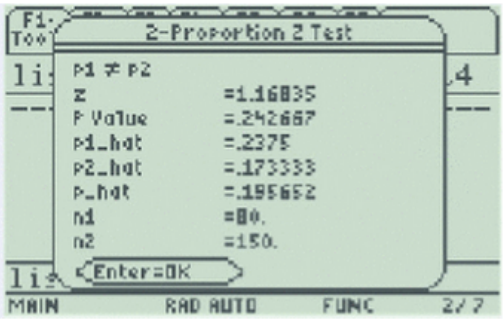
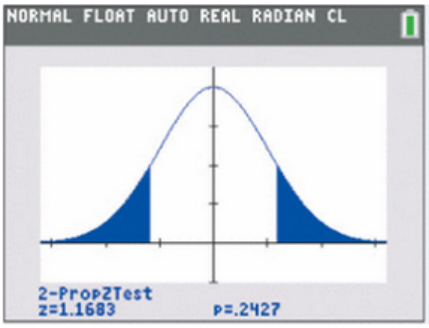


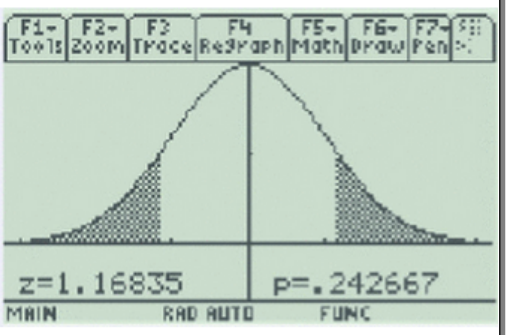
- If you select "Calculate" and press ENTER, you will see that the test statistic is $z = 1.168$ and the P-value is 0.2427. Do you see the combined proportion of students who didn't eat breakfast? It's the value labeled $\hat{p}$, 0.1957.





- If you select the "Draw" option, you will see the screen shown here.





Hearing loss
Are teenagers going deaf? In a study of 3000 randomly selected teenagers in 1988–1994, 15% showed some hearing loss. In a similar study of 1800 teenagers in 2005–2006, 19.5% showed some hearing loss. (These data are reported in *Arizona Daily Star*, August 18, 2010.)
Problem:
(a) Do these data give convincing evidence that the proportion of all teens with hearing loss has increased?
(b) Between the two studies, Apple introduced the iPod. If the results of the test are statistically significant, can we blame iPods for the increased hearing loss in teenagers?

(Check Assumptions)

- 2prop Z test

- $z = -4.05$ $pval \approx 0 < \alpha = .05$

Reject H_0
We have convincing evidence that the prop. of teens w/ hearing loss has increased

Inference for Experiments

Many important statistical results come from randomized comparative experiments. Defining the parameters in experimental settings is more challenging.

•Most experiments on people use recruited volunteers as subjects.

•When subjects are not randomly selected, researchers cannot generalize the results of an experiment to some larger populations of interest.

•Researchers can draw cause-and-effect conclusions that apply to people like those who took part in the experiment.

•Unless the experimental units are randomly selected, we don't need to check the 10% condition when performing inference about an experiment.

Cash for quitters
In an effort to reduce health care costs, General Motors sponsored a study to help employees stop smoking. In the study, half of the subjects were randomly assigned to receive up to $750 for quitting smoking for a year while the other half were simply encouraged to use traditional methods to stop smoking. None of the 878 volunteers knew that there was a financial incentive when they signed up. At the end of one year, 15% of those in the financial rewards group had quit smoking while only 5% in the traditional group had quit smoking. Do the results of this study give convincing evidence that a financial incentive helps people quit smoking? (These data are reported in *Arizona Daily Star,* February 11, 2009.)

$H_0: P_F - P_T = 0 \quad H_a: P_F - P_T > 0$

$P_F =$ ~~~
$P_T =$ ~~~

random assignment ✓
Large counts ✓

$x_1 = 66$
$n_1 = 439$
$x_2 = 22$
$n_2 = 439$

2 prop z test
$z = 4.94$ pval $\approx 0 < \alpha = .05$

Reject H_0

We have convincing evidence that employees like these have a higher quitting rate when given financial incentive.

CHECK YOUR UNDERSTANDING

To study the long-term effects of preschool programs for poor children, researchers designed an experiment. They recruited 123 children who had never attended preschool from low-income families in Michigan. Researchers randomly assigned 62 of the children to attend preschool (paid for by the study budget) and the other 61 to serve as a control group who would not go to preschool. One response variable of interest was the need for social services as adults. Over a 10-year period, 38 children in the preschool group and 49 in the control group have needed social services.[4]

Does this study provide convincing evidence that preschool reduces the later need for social services? Justify your answer.

State: $H_0: p_1 - p_2 = 0 \quad H_a: p_1 - p_2 > 0$

p_1 = true prop. of children like those in the study who did not attend preschool & used social services later

p_2 = true prop. of children like those in the study who did attend preschool & used social services later

Plan 2 prop z test for $p_1 - p_2$
Random ✓ random assignment
Large counts ✓

	S.S.	No S.S.	total
Pre-school (p_2)	38	24	62
No P.S. (p_1)	49	12	61

all ≥ 10

Do: $z = 2.32 \quad pval = .0102$

Conclude: pval $< \alpha = .05$, Reject H_0
Convincing evidence that the true prop. like those in the study who do not attend attend preschool that use Social Services later is greater