5-3 The Rational Numbers

- define rational numbers
- convert between improper fractions and mixed numbers
- reduce to lowest terms
- multiply and divide fractions
- add and subtract fractions

A **rational number** is any number that can be written as a fraction in form *a/b* where a and *b* are both integers (and *b* is not zero). The integer *a* is called the numerator and *b* is called the denominator.

- When the numerator of a fraction has a smaller absolute value than the denominator, we call that a **proper fraction**.
- When the absolute value of the numerator is larger than or equal to the denominator, we call that an **improper fraction**.

- A **mixed number** consists of a whole number and a fraction.

Ex 1) Write $\frac{17}{6}$ as a mixed number

$2\frac{5}{6}$

$6\overline{)17}$ 2, -12, 5

Try this one Write $\frac{23}{7}$ as a mixed number

$3\frac{2}{7}$

$7\overline{)23}$ 3, -21, 2

$\frac{812}{5}$

$= 162\frac{2}{5}$

$5\overline{)812}$ 162, -5, 31, -30, 12, -10, 2

Ex 2) Write $7\frac{2}{5}$ as an improper fraction $\frac{37}{5}$

$7(5)+2=37$

Try this one Write $10\frac{1}{4}$ as an improper fraction $\frac{41}{4}$

$10(4)+1=41$

Try these:

Write as a mixed number.

a) $43/6$ $6\overline{)43}$ -42 1 $7\frac{1}{6}$

b) $\frac{179}{8}$ $8\overline{)179}$ -16 19 -16 3 $22\frac{3}{8}$

Write as an improper fraction.

a) $3\frac{7}{9} = \frac{34}{9}$ $3(9)+7=34$

b) $153\frac{1}{6}$ $\frac{919}{6}$

$153 \times 6 = 918$ $+1$ 919

Ex 3) Reduce to lowest terms $\frac{18}{24}$

$\frac{18\div2}{24\div2}=\frac{9\div3}{12\div3}=\frac{3}{4}$ OR $\frac{18\div6}{24\div6}=\frac{3}{4}$

Try this one $\frac{56}{64}$

$\frac{56\div8}{64\div8}=\frac{7}{8}$

Ex 4) Rewriting a fraction with a larger denominator (equivalent fractions)

a) $\frac{3}{8}=\frac{?}{32}$

$\frac{3\cdot4}{8\cdot4}=\frac{12}{32}$

b) $\frac{5}{4}=\frac{?}{56}$

$\frac{5\cdot14}{4\cdot14}=\frac{70}{56}$

Try this one

a) $\frac{3}{8}=\frac{?}{24}$

b) $\frac{7}{13}=\frac{?}{65}$

c) $\frac{5}{9}=\frac{?}{99}$

d) $\frac{4}{5}=\frac{?}{50}$

HW 5.3 Part 1
#10-30
Ewen

EXAMPLE 1 Writing an Improper Fraction as a Mixed Number

Write $\frac{17}{6}$ as a mixed number.

SOLUTION

First, divide 17 by 6 using long division:

$$\begin{array}{r} 2 \\ 6\overline{)17} \\ -12 \\ \hline 5 \end{array}$$

The quotient is 2, with remainder 5.

That tells us that $\frac{17}{6} = 2 + \frac{5}{6}$

In mixed number form, we write $\frac{17}{6} = 2\frac{5}{6}$

EXAMPLE 2 Writing a Mixed Number as an Improper Fraction

Write $7\frac{2}{5}$ as an improper fraction.

SOLUTION

Step 1 Multiply the whole number part (7 in this case) by the denominator (5), then add the numerator (2):

→ $7 \cdot 5 + 2 = 35 + 2 = 37$

Step 2 Write as a fraction with the result of step 1 as numerator, and the original denominator of the fractional part.

$$7\frac{2}{5} = \frac{37}{5}$$

$6\frac{5}{7} = \frac{47}{7}$

$6(7)+5=47$

$3\frac{1}{9} = \frac{28}{9}$

Facts About Fractions

Since fractions can be thought of as the division of two integers, we can take advantage of rules for division of signed numbers to address the sign of a fraction.

3/5

Different Signs Negative Value

$$\frac{-3}{5} = \frac{3}{-5} = -\frac{3}{5}$$

You can **reduce a fraction to lowest terms** by dividing the numerator and denominator by the same number.

$$\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

In this case, 4/8 and 1/2 are **equivalent fractions**, which means they represent the same number.

EXAMPLE 3 Reducing a Fraction to Lowest Terms

Reduce $\frac{18}{24}$ to lowest terms.

SOLUTION

Both the numerator and denominator can be divided by 2 with no remainder:

$$\frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{9}{12}$$

Before we congratulate ourselves, notice that there's still more that can be done: each of 9 and 12 is divisible by 3!

$$\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

Now the fraction is in lowest terms because 3 and 4 have no common divisors.

EXAMPLE 3 Reducing a Fraction to Lowest Terms

SOLUTION CONTINUED

We could have accomplished the previous answer in one step by finding the greatest common factor (GCF) of 18 and 24, which is 6, then dividing the numerator and denominator by it:

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

Either method is acceptable.

$\frac{3}{4} \cdot \frac{4}{4} = \frac{?}{16}$ $\frac{12}{16}$

$\frac{2}{5} \cdot \frac{20}{20} = \frac{?}{100}$ $\frac{40}{100}$

EXAMPLE 4 Rewriting a Fraction with a Larger Denominator

$\frac{3}{8} = \frac{?}{32}$

SOLUTION

(a) To change the denominator from 8 to 32, we have to multiply by 4. So to make the fractions equivalent, we also need to multiply the numerator by 4. The equivalent fraction is

$$\frac{3 \times 4}{8 \times 4} = \frac{12}{32}$$

(b) It may be a bit harder to notice what number we need to multiply 4 by to get 56, so we divide $56 \div 4 = 14$. The number we need to multiply the numerator and denominator by is 14.

$$\frac{5 \times 14}{4 \times 14} = \frac{70}{56}$$

To **multiply fractions**, multiply the numerators and denominators separately.

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$$

$$\frac{-5}{9}=-\frac{5}{9}=\frac{5}{-9}$$

Ex 5

a) $\frac{5}{8}*\frac{3}{5}$

$\frac{15\cdot 3}{8\cdot 5}=\frac{3}{8}$

$\frac{15\div 5}{40\div 5}=\frac{3}{8}$

b) $-\frac{5}{9}*\frac{3}{11}$

$-\frac{5\cdot 3}{9\cdot 11}$

$=\frac{-5}{33}=-\frac{5}{33}$

c) $1\frac{3}{4}*2\frac{2}{5}=\frac{7}{4}\cdot\frac{12}{5}$

$\frac{7\cdot 12}{14\cdot 5}=\frac{21}{5}$

$=4\frac{1}{5}$

Try this one

a) $\frac{2}{7}\cdot\left(-\frac{21}{8}\right)$

$-\frac{12\cdot 21}{14\cdot 8}=-\frac{3}{4}$

b) $\frac{11}{9}*\frac{12}{55}$

$\frac{11\cdot 12}{9\cdot 55}$

$\frac{4}{15}$

c) $3\frac{1}{2}*2\frac{2}{3}=\frac{7}{2}\cdot\frac{8}{3}$

$\frac{7\cdot 8}{2\cdot 3}$

$\frac{28}{3}=9\frac{1}{3}$

To **divide** fractions, mutliply the first by the reciprocal of the second.

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}*\frac{d}{c}$$

flip the 2nd fraction and multiply

ex 6

a) $\frac{3}{4}\div\left(-\frac{5}{8}\right)=\frac{3}{4}\cdot-\frac{8}{5}$

$-\frac{3\cdot 8}{4\cdot 5}=\frac{-6}{5}=-1\frac{1}{5}$

b) $\frac{11}{2}\div\left(\frac{4}{9}\right)=\frac{11}{2}\cdot\frac{9}{4}$

$\frac{11\cdot 9}{2\cdot 4}=\frac{99}{8}$

Try this one

a) $-\frac{5}{3}\div\left(-\frac{10}{9}\right)=-\frac{5}{3}\cdot\frac{9}{10}$

$\frac{15\cdot 9}{13\cdot 10}=\frac{3}{2}$

$1\frac{1}{2}$

b) $\frac{14}{5}\div\left(\frac{3}{8}\right)=\frac{14}{5}\cdot\frac{8}{3}$

$\frac{112}{15}$

Story Problems:

Lamont and Marge are driving to a business conference. They stop for gas, and they estimate they have driven 3/5 of the distance. If the total trip is 285 miles, how many miles have they driven?

$\frac{3}{5} \cdot \frac{285}{1} = 171$ mi

According to the US Census Bureau, 3/10 of Americans without a high school diploma don't have health insurance. In a group of 250 high school dropouts, how many don't have insurance?

$\frac{3}{10} \cdot \frac{250}{1} = 75$ people

HW 5.3 part 2
p. 244:
35-41, 45
87-90, 95

Warm Up Answers

Mixed #

a) $8\frac{1}{3}$ b) $4\frac{2}{3}$ c) $45\frac{1}{3}$

Improper

a) $\frac{19}{4}$ b) $\frac{27}{4}$ c) $\frac{106}{11}$

Simplify

a) $4/5$ b) $-\frac{164}{15}$ $-10\frac{14}{15}$ c) $-\frac{45}{4}$ or $-11\frac{1}{4}$

a) $7/16$ b) $-9/10$ c) $-1/12$

b) $-2\frac{2}{3} \cdot 4\frac{1}{10}$

$-\frac{8}{3} \cdot \frac{41}{10}$ (4, 5)

$-\frac{164}{15}$

$-\frac{164}{15}$

c) $-1\frac{1}{4} \cdot 9$

$-\frac{5}{4} \cdot \frac{9}{1}$

$-\frac{45}{4}$

c) $\frac{1}{9} \div -1\frac{1}{3}$

$\frac{1}{9} \div -\frac{4}{3}$

$\frac{1}{\cancel{9}_{3}} \cdot -\frac{\cancel{3}^{1}}{4} = -\frac{1}{12}$

$-\frac{3}{36} = -\frac{1}{12}$

HW answers

35) 7/24

36) 1/10

37) 7/6 or 1 1/6

38) -35/92

39) 7/10

40) -315/256

41) -1/16

45) 3/4

87) 75 people

88) 113.1 mil people

89) 4580 adults

90) 190 mi

95) 1 1/4 c fl 1/3 c sugar

$2\frac{1}{2} \cdot \frac{1}{2} = \frac{5}{2} \cdot \frac{1}{2} = \frac{5}{4}$

$\frac{2}{3} \cdot \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$

$300/60 = 5$ batches

$2\frac{1}{4} \cdot 5 = \frac{9}{4} \cdot \frac{5}{1} = \frac{45}{4} = 11\frac{1}{4}$ cups flour

$\frac{3}{4} \cdot \frac{5}{1} = \frac{15}{4} = 3\frac{3}{4}$ C of sugar & brown sugar

Adding & Subtracting Fractions

Adding/Subtracting with Common Denominators
To add or subtract two fractions with the same denominator, add or subtract the numerators, and keep the common denominator the same in your answer.

Same Denom.

1. add or subtract top
2. keep bottom the same
3. reduce

EXAMPLE 7 Adding & Subtracting Fractions with a Common Denominator

Find each sum or difference.

(a) $\frac{5}{12} + \frac{11}{12}$ (b) $\frac{7}{3} - \frac{4}{3}$

$\frac{5+11}{12}$ $\frac{7-4}{3}$

$\frac{16}{12}$ $\frac{3}{3}$

$\boxed{\frac{4}{3}}$ or $1\frac{1}{3}$ $\boxed{1}$

Your turn

d) $\frac{17}{9} + \frac{4}{9}$ e) $\frac{2}{15} - \frac{11}{15}$

$\frac{7}{3}$ or $2\frac{1}{3}$ $-\frac{3}{5}$

Adding/Subtracting with Different Denominators

Step 1 Find the least common multiple of the denominators. (This is usually called the **least common denominator**, or **LCD**.)

Step 2 Rewrite each fraction as an equivalent fraction with denominator equal to the LCD.

Step 3 Add or subtract.

① find common multiple
② rewrite fractions w/ same denon.
③ add or sub
④ reduce

EXAMPLE 8 Adding & Subtracting Fractions

Find each sum or difference.

(a) $\frac{1}{4}+\frac{5}{6}$ (b) $\frac{4}{9}-\frac{2}{5}$

a) $\frac{1}{4}=\frac{6}{24}$ $\frac{5}{6}=\frac{20}{24}$

$\frac{6+20}{24}$

$\frac{26}{24}=\frac{13}{12}$

b) $\frac{4}{9}=\frac{20}{45}$ $\frac{2}{5}=\frac{18}{45}$

$\frac{20-18}{45}=\frac{2}{45}$

Your turn

d) $-\frac{3}{8}+\frac{5}{6}$

$\frac{-18}{48}+\frac{40}{48}$

$\frac{22}{48}=\frac{11}{24}$

e) $\frac{9}{10}-\frac{1}{2}$

$\frac{9}{10}-\frac{5}{10}$

$\frac{4}{10}=\frac{2}{5}$

5.3 Part 3

p.244 #31-34, 42-44, 47, 48

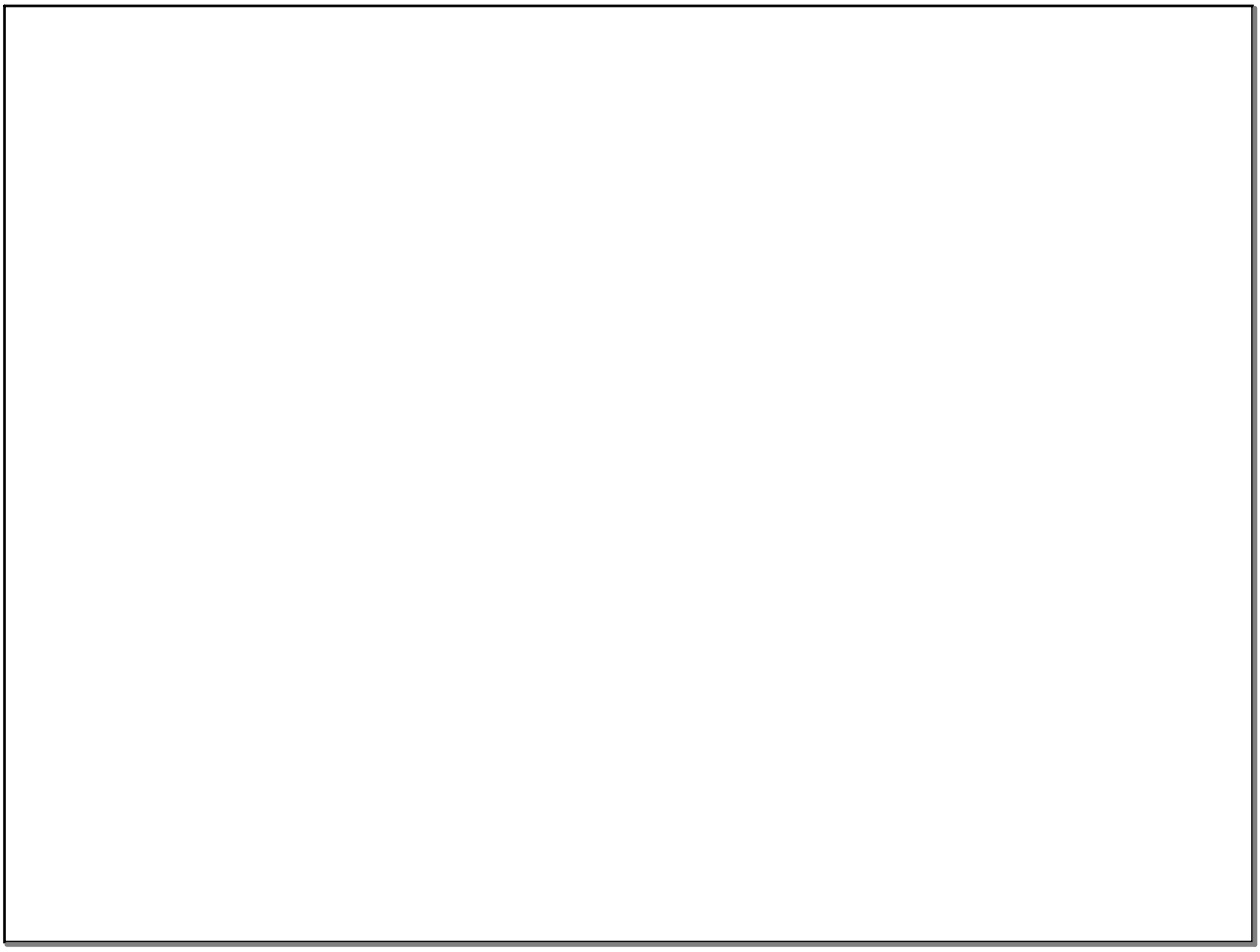
f) $5\frac{3}{4}-4\frac{1}{3}$

(c) $2\frac{1}{2}+3\frac{1}{4}$

In her final 2 days of preparation for a triathlon, Cat hopes to swim, run, and bike a total of 50 miles. She works out at a nearby state park with a swimming quarry and running/biking trail. One lap in the quarry is $\frac{1}{4}$ mile and the trail is $3\frac{1}{4}$ miles.

The first day, Cat swims six laps, runs the trail twice, and bikes it five times. How many more miles does she need to cover the second day?

Your turn: If Cat swims 3 laps and runs and bikes 4 laps each on the second day, will she reach her goal? How much is she over or under by?

The Daytona International Speedway is 5/2 miles long and the Iowa Speedway is 7/8 miles long. How much longer is a 250-lap race is Daytona?

An estate is divided among five people. The first person received 1/8 of the estate. The next two people received 1/5 of the estate. The fourth person received 1/10 of the estate. What fractional part of the estate did the last person receive?