



## Section 8.3 Exercises

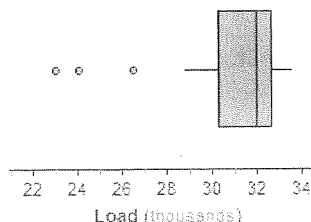
pg 513 55. **Critical values** What critical value  $t^*$  from Table B would you use for a confidence interval for the population mean in each of the following situations?

- (a) A 95% confidence interval based on  $n = 10$  randomly selected observations
- (b) A 99% confidence interval from an SRS of 20 observations
- (c) A 90% confidence interval based on a random sample of 77 individuals

56. **Critical values** What critical value  $t^*$  from Table B should be used for a confidence interval for the population mean in each of the following situations?

- (a) A 90% confidence interval based on  $n = 12$  randomly selected observations
- (b) A 95% confidence interval from an SRS of 30 observations
- (c) A 99% confidence interval based on a random sample of size 58

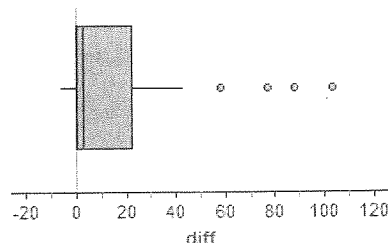
57. **Pulling wood apart** How heavy a load (pounds) is needed to pull apart pieces of Douglas fir 4 inches long and 1.5 inches square? A random sample of 20 similar pieces of Douglas fir from a large batch was selected for a science class. The Fathom boxplot below shows the class's data. Explain why it would not be wise to use a  $t$  critical value to construct a confidence interval for the population mean  $\mu$ .



58. **Weeds among the corn** Velvetleaf is a particularly annoying weed in cornfields. It produces lots of seeds, and the seeds wait in the soil for years until conditions are right for sprouting. How many seeds do velvetleaf plants produce? The Fathom histogram below shows the counts from a random sample of 28 plants that came up in a cornfield when no herbicide was used.<sup>20</sup> Explain why it would not be wise to use a  $t$  critical value to construct a confidence interval for the mean number of seeds  $\mu$  produced by velvetleaf plants.

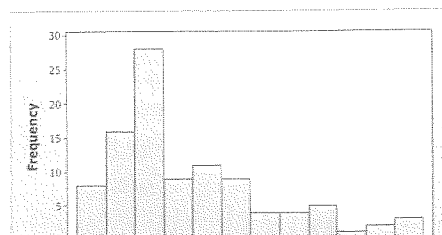
pg 516 59. **Should we use  $t$ ?** Determine whether we can safely use a  $t^*$  critical value to calculate a confidence interval for the population mean in each of the following settings.

- (a) We collect data from a random sample of adult residents in a state. Our goal is to estimate the overall percent of adults in the state who are college graduates.
- (b) The coach of a college men's basketball team records the resting heart rates of the 15 team members. We use these data to construct a confidence interval for the mean resting heart rate of all male students at this college.
- (c) Do teens text more than they call? To find out, an AP<sup>®</sup> Statistics class at a large high school collected data on the number of text messages and calls sent or received by each of 25 randomly selected students. The Fathom boxplot below displays the difference (texts – calls) for each student.



60. **Should we use  $t$ ?** Determine whether we can safely use a  $t^*$  critical value to calculate a confidence interval for the population mean in each of the following settings.

- (a) We want to estimate the average age at which U.S. presidents have died. So we obtain a list of all U.S. presidents who have died and their ages at death.
- (b) How much time do students spend on the Internet? We collect data from the 32 members of our AP<sup>®</sup> Statistics class and calculate the mean amount of time that each student spent on the Internet yesterday.
- (c) Judy is interested in the reading level of a medical journal. She records the length of a random sample of 100 words. The Minitab histogram below displays the data.



62. **Travel time to work** A study of commuting times reports the travel times to work of a random sample of 20 employed adults in New York State. The mean is  $\bar{x} = 31.25$  minutes, and the standard deviation is  $s_x = 21.88$  minutes. What is the standard error of the mean? Interpret this value in context.

63. **Willows in Yellowstone** Writers in some fields summarize data by giving  $\bar{x}$  and its standard error rather than  $\bar{x}$  and  $s_x$ . Biologists studying willow plants in Yellowstone National Park reported their results in a table with columns labeled  $\bar{x} \pm \text{SE}$ . The table entry for the heights of willow plants (in centimeters) in one region of the park was  $61.55 \pm 19.03$ .<sup>21</sup> The researchers measured a total of 23 plants.

- (a) Find the sample standard deviation  $s_x$  for these measurements. Show your work.
- (b) A hasty reader believes that the interval given in the table is a 95% confidence interval for the mean height of willow plants in this region of the park. Find the actual confidence level for the given interval.

64. **Blink** When two lights close together blink alternately, we “see” one light moving back and forth if the time between blinks is short. What is the longest interval of time between blinks that preserves the illusion of motion? Ask subjects to turn a knob that slows the blinking until they “see” two lights rather than one light moving. A report gives the results in the form “mean plus or minus the standard error of the mean.”<sup>22</sup> Data for 12 subjects are summarized as  $251 \pm 45$  (in milliseconds).

- (a) Find the sample standard deviation  $s_x$  for these measurements. Show your work.
- (b) A hasty reader believes that the interval given in the report is a 95% confidence interval for the population mean. Find the actual confidence level for the given interval.

65. **Bone loss by nursing mothers** Breast-feeding mothers secrete calcium into their milk. Some of the calcium may come from their bones, so mothers may lose bone mineral. Researchers measured the percent change in bone mineral content (BMC) of the spines of 47 randomly selected mothers during three months of breast-feeding.<sup>23</sup> The mean change in BMC was  $-3.587\%$  and the standard deviation was  $2.506\%$ .

- (a) Construct and interpret a 99% confidence interval to estimate the mean percent change in BMC in the population.
- (b) Based on your interval from part (a), do these data give good evidence that on the average nursing mothers lose bone mineral? Explain.

66. **Reading scores in Atlanta** The Trial Urban District Assessment (TUDA) is a government-sponsored study of student achievement in large urban school districts. TUDA gives a reading test scored from 0 to 500. A score of 243 is a “basic” reading level and a score of 281 is “proficient.” Scores for a random sample of

1470 eighth-graders in Atlanta had  $\bar{x} = 240$  with standard deviation 42.17.<sup>24</sup>

- (a) Calculate and interpret a 99% confidence interval for the mean score of all Atlanta eighth-graders.
- (b) Based on your interval from part (a), is there good evidence that the mean for all Atlanta eighth-graders is less than the basic level? Explain.

67. **Men and muscle** Ask young men to estimate their own degree of body muscle by choosing from a set of 100 photos. Then ask them to choose what they believe women prefer. The researchers know the actual degree of muscle, measured as kilograms per square meter of fat-free mass, for each of the photos. They can therefore measure the difference between what a subject thinks women prefer and the subject’s own self-image. Call this difference the “muscle gap.” Here are summary statistics for the muscle gap from a random sample of 200 American and European young men:  $\bar{x} = 2.35$  and  $s_x = 2.5$ .<sup>25</sup>

- (a) Calculate and interpret a 95% confidence interval for the mean size of the muscle gap for the population of American and European young men.
- (b) A graph of the sample data is strongly skewed to the right. Explain why this information does not invalidate the interval you calculated in part (a).

68. **A big-toe problem** A bunion on the big toe is fairly uncommon in youth and often requires surgery. Doctors used X-rays to measure the angle (in degrees) of deformity on the big toe in a random sample of 37 patients under the age of 21 who came to a medical center for surgery to correct a bunion. The angle is a measure of the seriousness of the deformity. For these 37 patients, the mean angle of deformity was 24.76 degrees and the standard deviation was 6.34 degrees. A dotplot of the data revealed no outliers or strong skewness.<sup>26</sup>

- (a) Construct and interpret a 90% confidence interval for the mean angle of deformity in the population of all such patients.
- (b) Researchers omitted one patient with a deformity angle of 50 degrees from the analysis due to a measurement issue. What effect would including this outlier have on the confidence interval in part (a)? Justify your answer without doing any calculations.

69. **Give it some gas!** Computers in some vehicles calculate various quantities related to performance. One of these is fuel efficiency, or gas mileage, usually expressed as miles per gallon (mpg). For one vehicle equipped in this way, the miles per gallon were recorded each time the gas tank was filled and the computer was then reset.<sup>27</sup> Here are the mpg values for a random sample of 20 of these records:

15.8	13.6	15.6	19.1	22.4	15.6	22.5	17.2	19.4	22.6
19.4	18.0	14.6	18.7	21.0	14.8	22.6	21.5	14.3	20.9

Construct and interpret a 95% confidence interval for the mean fuel efficiency  $\mu$  for this vehicle.



70. **Vitamin C content** Several years ago, the U.S. Agency for International Development provided 238,300 metric tons of corn-soy blend (CSB) for emergency relief in countries throughout the world. CSB is a highly nutritious, low-cost fortified food. As part of a study to evaluate appropriate vitamin C levels in this food, measurements were taken on samples of CSB produced in a factory.<sup>28</sup> The following data are the amounts of vitamin C, measured in milligrams per 100 grams (mg/100 g) of blend, for a random sample of size 8 from one production run:

26	31	23	22	11	22	14	31
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Construct and interpret a 95% confidence interval for the mean amount of vitamin C  $\mu$  in the CSB from this production run.

71. **Paired tires** Researchers were interested in comparing two methods for estimating tire wear. The first method used the amount of weight lost by a tire. The second method used the amount of wear in the grooves of the tire. A random sample of 16 tires was obtained. Both methods were used to estimate the total distance traveled by each tire. The table below provides the two estimates (in thousands of miles) for each tire.<sup>29</sup>
- (a) Construct and interpret a 95% confidence interval for the mean difference  $\mu$  in the estimates from these two methods in the population of tires.
- (b) Does your interval in part (a) give convincing evidence of a difference in the two methods of estimating tire wear? Justify your answer.

Tire	Weight	Groove	Diff.	Tire	Weight	Groove	Diff.
1	45.9	35.7	10.2	9	30.4	23.1	7.3
2	41.9	39.2	2.7	10	27.3	23.7	3.6
3	37.5	31.1	6.4	11	20.4	20.9	-0.5
4	33.4	28.1	5.3	12	24.5	16.1	8.4
5	31.0	24.0	7.0	13	20.9	19.9	1.0
6	30.5	28.7	1.8	14	18.9	15.2	3.7
7	30.9	25.9	5.0	15	13.7	11.5	2.2
8	31.9	23.3	8.6	16	11.4	11.2	0.2

72. **Water** Trace metals found in wells affect the taste of drinking water, and high concentrations can pose a health risk. Researchers measured the concentration of zinc (in milligrams/liter) near the top and the bottom of 10 randomly selected wells in a large region. The data are provided in the table below.<sup>30</sup>
- (a) Construct and interpret a 95% confidence interval for the mean difference  $\mu$  in the zinc concentrations from these two locations in the wells.
- (b) Does your interval in part (a) give convincing evidence of a difference in zinc concentrations at the top and bottom of wells in the region? Justify your answer.

Well:	1	2	3	4	5	6	7	8	9	10
Bottom:	0.430	0.266	0.567	0.531	0.707	0.716	0.651	0.589	0.469	0.723
Top:	0.415	0.238	0.390	0.410	0.605	0.609	0.632	0.523	0.411	0.612
Difference:	0.015	0.028	0.177	0.121	0.102	0.107	0.019	0.066	0.058	0.111

73. **Estimating BMI** The body mass index (BMI) of all American young women is believed to follow a Normal distribution with a standard deviation of about 7.5. How large a sample would be needed to estimate the mean BMI  $\mu$  in this population to within  $\pm 1$  with 99% confidence? Show your work.
74. **The SAT again** High school students who take the SAT Math exam a second time generally score higher than on their first try. Past data suggest that the score increase has a standard deviation of about 50 points. How large a sample of high school students would be needed to estimate the mean change in SAT score to within 2 points with 95% confidence? Show your work.

*Multiple choice: Select the best answer for Exercises 75 to 78.*

75. One reason for using a  $t$  distribution instead of the standard Normal curve to find critical values when calculating a level  $C$  confidence interval for a population mean is that
- (a)  $z$  can be used only for large samples.  
 (b)  $z$  requires that you know the population standard deviation  $\sigma$ .  
 (c)  $z$  requires that you can regard your data as an SRS from the population.  
 (d)  $z$  requires that the sample size is at most 10% of the population size.  
 (e) a  $z$  critical value will lead to a wider interval than a  $t$  critical value.
76. You have an SRS of 23 observations from a large population. The distribution of sample values is roughly symmetric with no outliers. What critical value would you use to obtain a 98% confidence interval for the mean of the population?
- (a) 2.177 (b) 2.183 (c) 2.326 (d) 2.500 (e) 2.508
77. A quality control inspector will measure the salt content (in milligrams) in a random sample of bags of potato chips from an hour of production. Which of the following would result in the smallest margin of error in estimating the mean salt content  $\mu$ ?
- (a) 90% confidence;  $n = 25$   
 (b) 90% confidence;  $n = 50$   
 (c) 95% confidence;  $n = 25$   
 (d) 95% confidence;  $n = 50$   
 (e)  $n = 100$  at any confidence level
78. Scientists collect data on the blood cholesterol levels (milligrams per deciliter of blood) of a random sample of 24 laboratory rats. A 95% confidence interval for the mean blood cholesterol level  $\mu$  is 80.2 to 89.8. Which of the following would cause the most worry about the validity of this interval?
- (a) There is a clear outlier in the data.  
 (b) A stemplot of the data shows a mild right skew.

- (c) You do not know the population standard deviation  $\sigma$ .  
 (d) The population distribution is not exactly Normal.  
 (e) None of these are a problem when using a  $t$  interval.

79. **Watching TV** (6.1, 7.3) Choose a young person (aged 19 to 25) at random and ask, "In the past seven days, how many days did you watch television?" Call the response  $X$  for short. Here is the probability distribution for  $X$ :<sup>31</sup>

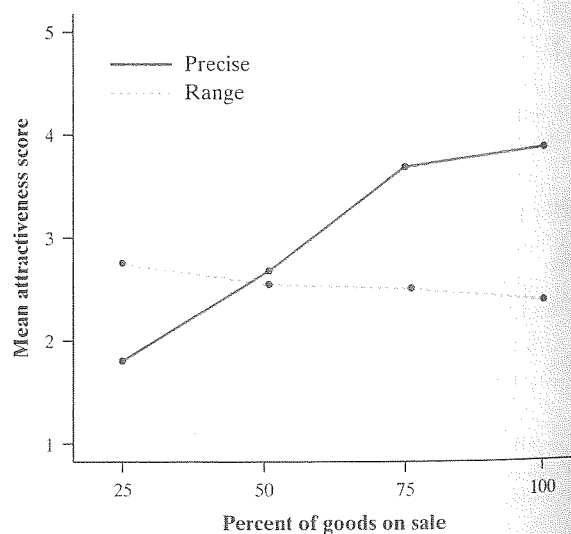
Days:	0	1	2	3	4	5	6	7
Probability:	0.04	0.03	0.06	0.08	0.09	0.08	0.05	???

- (a) What is the probability that  $X = 7$ ? Justify your answer.  
 (b) Calculate the mean of the random variable  $X$ . Interpret this value in context.  
 (c) Suppose that you asked 100 randomly selected young people (aged 19 to 25) to respond to the question and found that the mean  $\bar{x}$  of their responses was 4.96. Would this result surprise you? Justify your answer.
80. **Price cuts** (4.2) Stores advertise price reductions to attract customers. What type of price cut is most attractive? Experiments with more than one factor allow insight into interactions between the factors. A study of the attractiveness of advertised price discounts had two factors: percent of all foods on sale (25%, 50%, 75%, or 100%) and whether the discount was stated precisely (as in, for example, "60% off") or as a range (as in "40% to 70% off"). Subjects rated the attractiveness of the sale on a scale of 1 to 7.

- (a) Describe a completely randomized design using 200 student subjects.  
 (b) Explain how you would use the partial table of random digits below to assign subjects to treatment groups. Then use your method to select the first 3 subjects for one of the treatment groups. Show your work clearly on your paper.

45740 41807 65561 33302 07051 93623 18132 09547  
 12975 13258 13048 45144 72321 81940 00360 02428

- (c) The figure below shows the mean ratings for the eight treatments formed from the two factors.<sup>32</sup> Based on these results, write a careful description of how percent on sale and precise discount versus range of discounts influence the attractiveness of a sale.



## FRAPPY! Free Response AP<sup>®</sup> Problem, Yay!

The following problem is modeled after actual AP<sup>®</sup> Statistics exam free response questions. Your task is to generate a complete, concise response in 15 minutes.

*Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.*

Members at a popular fitness club currently pay a \$40 per month membership fee. The owner of the club wants to raise the fee to \$50 but is concerned that some members will leave the gym if the fee increases. To investigate, the owner plans to survey a random sample of the club members and construct a 95% confidence interval for the proportion of all members who would quit if the fee was raised to \$50.

- (a) Explain the meaning of "95% confidence" in the context of the study.  
 (b) After the owner conducted the survey, he calculated the confidence interval to be  $0.18 \pm 0.075$ .

Interpret this interval in the context of the study.

- (c) According to the club's accountant, the fee increase will be worthwhile if fewer than 20% of the members quit. According to the interval from part (b), can the owner be confident that the fee increase will be worthwhile? Explain.  
 (d) One of the conditions for calculating the confidence interval in part (b) is that  $n\hat{p} \geq 10$  and  $n(1 - \hat{p}) \geq 10$ . Explain why it is necessary to check this condition.

After you finish, you can view two example solutions on the book's Web site ([www.whfreeman.com/tps5e](http://www.whfreeman.com/tps5e)). Determine whether you think each solution is "complete," "substantial," "developing," or "minimal." If the solution is not complete, what improvements would you suggest to the student who wrote it? Finally, your teacher will provide you with a scoring rubric. Score your response and note what, if anything, you would do differently to improve your own score.