



4. Assume that the actual proportion of unsuitable batteries produced that day is $p = 0.27$. Describe the shape, center, and spread of the sampling distribution of \hat{p} for random samples of 50 batteries. Justify your answers.

For the random sample of 50 batteries, the sample proportion with lifetimes less than 16.5 hours was $\hat{p} = 0.32$.

5. Find the probability of obtaining a random sample of 50 batteries in which 32% or more of the batteries are unsuitable if $p = 0.27$. Show your work. Based on your answer, should the entire batch of batteries be shipped to customers? Why or why not?

Section 7.3

Summary

- When we want information about the population mean μ for some variable, we often take an SRS and use the sample mean \bar{x} to estimate the unknown parameter μ . The **sampling distribution** of \bar{x} describes how the statistic \bar{x} varies in *all* possible samples of the same size from the population.
- The **mean** of the sampling distribution is μ , so \bar{x} is an unbiased estimator of μ .
- The **standard deviation** of the sampling distribution of \bar{x} is σ/\sqrt{n} for an SRS of size n if the population has standard deviation σ . That is, averages are less variable than individual observations. This formula can be used if the population is at least 10 times as large as the sample (*10% condition*).
- Choose an SRS of size n from a population with mean μ and standard deviation σ . If the population distribution is Normal, then so is the sampling distribution of the sample mean \bar{x} . If the population distribution is not Normal, the **central limit theorem (CLT)** states that when n is large, the sampling distribution of \bar{x} is approximately Normal.
- We can use a Normal distribution to calculate approximate probabilities for events involving \bar{x} whenever the *Normal/Large Sample condition* is met:
 - If the population distribution is Normal, so is the sampling distribution of \bar{x} .
 - If $n \geq 30$, the CLT tells us that the sampling distribution of \bar{x} will be approximately Normal in most cases.

Section 7.3

Exercises

49. **Songs on an iPod** David's iPod has about 10,000 songs. The distribution of the play times for these songs is heavily skewed to the right with a mean of 225 seconds and a standard deviation of 60 seconds.

Suppose we choose an SRS of 10 songs from this population and calculate the mean play time \bar{x} of these songs. What are the mean and the standard deviation of the sampling distribution of \bar{x} ? Explain.

50. **Making auto parts** A grinding machine in an auto parts plant prepares axles with a target diameter $\mu = 40.125$ millimeters (mm). The machine has some variability, so the standard deviation of the diameters is $\sigma = 0.002$ mm. The machine operator inspects a random sample of 4 axles each hour for quality control purposes and records the sample mean diameter \bar{x} . Assuming that the process is working properly, what are the mean and standard deviation of the sampling distribution of \bar{x} ? Explain.
51. **Songs on an iPod** Refer to Exercise 49. How many songs would you need to sample if you wanted the standard deviation of the sampling distribution of \bar{x} to be 30 seconds? Justify your answer.
52. **Making auto parts** Refer to Exercise 50. How many axles would you need to sample if you wanted the standard deviation of the sampling distribution of \bar{x} to be 0.0005 mm? Justify your answer.
53. **Larger sample** Suppose that the blood cholesterol level of all men aged 20 to 34 follows the Normal distribution with mean $\mu = 188$ milligrams per deciliter (mg/dl) and standard deviation $\sigma = 41$ mg/dl.
- Choose an SRS of 100 men from this population. Describe the sampling distribution of \bar{x} .
 - Find the probability that \bar{x} estimates μ within ± 3 mg/dl. (This is the probability that \bar{x} takes a value between 185 and 191 mg/dl.) Show your work.
 - Choose an SRS of 1000 men from this population. Now what is the probability that \bar{x} falls within ± 3 mg/dl of μ ? Show your work. In what sense is the larger sample “better”?
54. **Dead battery?** A car company has found that the lifetime of its batteries varies from car to car according to a Normal distribution with mean $\mu = 48$ months and standard deviation $\sigma = 8.2$ months. The company installs a new brand of battery on an SRS of 8 cars.
- If the new brand has the same lifetime distribution as the previous type of battery, describe the sampling distribution of the mean lifetime \bar{x} .
 - The average life of the batteries on these 8 cars turns out to be $\bar{x} = 42.2$ months. Find the probability that the sample mean lifetime is 42.2 months or less if the lifetime distribution is unchanged. What conclusion would you draw?
55. **Bottling cola** A bottling company uses a filling machine to fill plastic bottles with cola. The bottles are supposed to contain 300 milliliters (ml). In fact, the contents vary according to a Normal distribution with mean $\mu = 298$ ml and standard deviation $\sigma = 3$ ml.
- What is the probability that a randomly selected bottle contains less than 295 ml? Show your work.
 - What is the probability that the mean contents of six randomly selected bottles are less than 295 ml? Show your work.
56. **Cereal** A company's cereal boxes advertise 9.65 ounces of cereal. In fact, the amount of cereal in a randomly selected box follows a Normal distribution with mean $\mu = 9.70$ ounces and standard deviation $\sigma = 0.03$ ounces.
- What is the probability that a randomly selected box of the cereal contains less than 9.65 ounces of cereal? Show your work.
 - Now take an SRS of 5 boxes. What is the probability that the mean amount of cereal \bar{x} in these boxes is 9.65 ounces or less? Show your work.
57. **What does the CLT say?** Asked what the central limit theorem says, a student replies, “As you take larger and larger samples from a population, the histogram of the sample values looks more and more Normal.” Is the student right? Explain your answer.
58. **What does the CLT say?** Asked what the central limit theorem says, a student replies, “As you take larger and larger samples from a population, the spread of the sampling distribution of the sample mean decreases.” Is the student right? Explain your answer.
59. **Songs on an iPod** Refer to Exercise 49.
- Explain why you cannot safely calculate the probability that the mean play time \bar{x} is more than 4 minutes (240 seconds) for an SRS of 10 songs.
 - Suppose we take an SRS of 36 songs instead. Explain how the central limit theorem allows us to find the probability that the mean play time is more than 240 seconds. Then calculate this probability. Show your work.
60. **Lightning strikes** The number of lightning strikes on a square kilometer of open ground in a year has mean 6 and standard deviation 2.4. The National Lightning Detection Network (NLDN) uses automatic sensors to watch for lightning in a random sample of 10 one-square-kilometer plots of land.
- What are the mean and standard deviation of the sampling distribution of \bar{x} , the sample mean number of strikes per square kilometer?
 - Explain why you cannot safely calculate the probability that $\bar{x} < 5$ based on a sample of size 10.
 - Suppose the NLDN takes a random sample of $n = 50$ square kilometers instead. Explain how the central limit theorem allows us to find the probability that the mean number of lightning strikes per square kilometer is less than 5. Then calculate this probability. Show your work.



61. **Airline passengers get heavier** In response to the increasing weight of airline passengers, the Federal Aviation Administration (FAA) told airlines to assume that passengers average 190 pounds in the summer, including clothes and carry-on baggage. But passengers vary, and the FAA did not specify a standard deviation. A reasonable standard deviation is 35 pounds. Weights are not Normally distributed, especially when the population includes both men and women, but they are not very non-Normal. A commuter plane carries 30 passengers.
- (a) Explain why you cannot calculate the probability that a randomly selected passenger weighs more than 200 pounds.
- (b) Find the probability that the total weight of 30 randomly selected passengers exceeds 6000 pounds. Show your work. (*Hint:* To apply the central limit theorem, restate the problem in terms of the mean weight.)
62. **How many people in a car?** A study of rush-hour traffic in San Francisco counts the number of people in each car entering a freeway at a suburban interchange. Suppose that this count has mean 1.5 and standard deviation 0.75 in the population of all cars that enter at this interchange during rush hours.
- (a) Could the exact distribution of the count be Normal? Why or why not?
- (b) Traffic engineers estimate that the capacity of the interchange is 700 cars per hour. Find the probability that 700 randomly selected cars at this freeway entrance will carry more than 1075 people. Show your work. (*Hint:* Restate this event in terms of the mean number of people \bar{x} per car.)
63. **More on insurance** An insurance company claims that in the entire population of homeowners, the mean annual loss from fire is $\mu = \$250$ and the standard deviation of the loss is $\sigma = \$1000$. The distribution of losses is strongly right-skewed: many policies have \$0 loss, but a few have large losses. An auditor examines a random sample of 10,000 of the company's policies. If the company's claim is correct, what's the probability that the average loss from fire in the sample is no greater than \$275? Show your work.
64. **Bad carpet** The number of flaws per square yard in a type of carpet material varies with mean 1.6 flaws per square yard and standard deviation 1.2 flaws per square yard. The population distribution cannot be Normal, because a count takes only whole-number values. An inspector studies a random sample of 200 square yards of the material, records the number of flaws found in each square yard, and calculates \bar{x} , the mean number of flaws per square yard inspected. Find the probability that the mean number of flaws exceeds 1.8 per square yard. Show your work.

Multiple choice: Select the best answer for Exercises 65 to 68.

65. Scores on the mathematics part of the SAT exam in a recent year were roughly Normal with mean 515 and standard deviation 114. You choose an SRS of 100 students and average their SAT Math scores. Suppose that you do this many, many times. Which of the following are the mean and standard deviation of the sampling distribution of \bar{x} ?
- (a) Mean = 515, SD = 114
- (b) Mean = 515, SD = $114/\sqrt{100}$
- (c) Mean = $515/100$, SD = $114/100$
- (d) Mean = $515/100$, SD = $114/\sqrt{100}$
- (e) Cannot be determined without knowing the 100 scores.
66. Why is it important to check the 10% condition before calculating probabilities involving \bar{x} ?
- (a) To reduce the variability of the sampling distribution of \bar{x} .
- (b) To ensure that the distribution of \bar{x} is approximately Normal.
- (c) To ensure that we can generalize the results to a larger population.
- (d) To ensure that \bar{x} will be an unbiased estimator of μ .
- (e) To ensure that the observations in the sample are close to independent.
67. A newborn baby has extremely low birth weight (ELBW) if it weighs less than 1000 grams. A study of the health of such children in later years examined a random sample of 219 children. Their mean weight at birth was $\bar{x} = 810$ grams. This sample mean is an *unbiased estimator* of the mean weight μ in the population of all ELBW babies, which means that
- (a) in all possible samples of size 219 from this population, the mean of the values of \bar{x} will equal 810.
- (b) in all possible samples of size 219 from this population, the mean of the values of \bar{x} will equal μ .
- (c) as we take larger and larger samples from this population, \bar{x} will get closer and closer to μ .
- (d) in all possible samples of size 219 from this population, the values of \bar{x} will have a distribution that is close to Normal.
- (e) the person measuring the children's weights does so without any error.