Section 6.2 Summary

- Adding a positive constant a to (subtracting a from) a random variable increases (decreases) the mean of the random variable by a but does not affect its standard deviation or the shape of its probability distribution.
- Multiplying (dividing) a random variable by a positive constant b multiplies (divides) the mean of the random variable by b and the standard deviation by b but does not change the shape of its probability distribution.
- A **linear transformation** of a random variable involves adding or subtracting a constant a, multiplying or dividing by a constant b, or both. We can write a linear transformation of the random variable X in the form $Y = a + bX$. The shape, center, and spread of the probability distribution of Y are as follows:

  Shape: Same as the probability distribution of X if $b > 0$.

  Center: $\mu_Y = a + b\mu_X$

  Spread: $\sigma_Y = |b|\sigma_X$
- If X and Y are *any* two random variables,

  $\mu_{X+Y} = \mu_X + \mu_Y$: The mean of the sum of two random variables is the sum of their means.

  $\mu_{X-Y} = \mu_X - \mu_Y$: The mean of the difference of two random variables is the difference of their means.
- If X and Y are **independent random variables**, then knowing the value of one variable tells you nothing about the value of the other. In that case, variances add:

  $\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$: The variance of the sum of two independent random variables is the sum of their variances.

  $\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$: The variance of the difference of two independent random variables is the sum of their variances.
- The sum or difference of independent Normal random variables follows a Normal distribution.

Section 6.2 Exercises

35. **Crickets** The length in inches of a cricket chosen at random from a field is a random variable X with mean 1.2 inches and standard deviation 0.25 inches. Find the mean and standard deviation of the length Y of a randomly chosen cricket from the field in centimeters. There are 2.54 centimeters in an inch.

36. **Men's heights** A report of the National Center for Health Statistics says that the height of a 20-year-old man chosen at random is a random variable H with mean 5.8 feet and standard deviation 0.24 feet. Find the mean and standard deviation of the height J of a randomly selected 20-year-old man in inches. There are 12 inches in a foot.

37. **Get on the boat!** A small ferry runs every half hour from one side of a large river to the other. The number of cars X on a randomly chosen ferry trip has the probability distribution shown below. You can check that $\mu_X = 3.87$ and $\sigma_X = 1.29$.

Cars:	0	1	2	3	4	5
Probability:	0.02	0.05	0.08	0.16	0.27	0.42

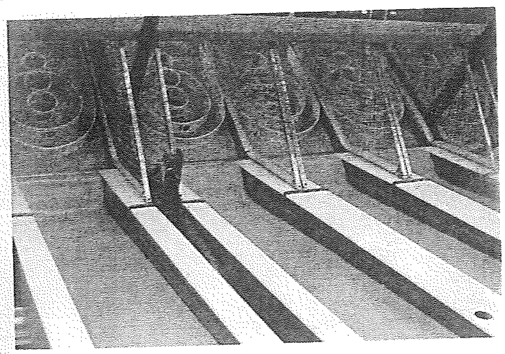
(a) The cost for the ferry trip is \$5. Make a graph of the probability distribution for the random variable M = money collected on a randomly selected ferry trip. Describe its shape.

(b) Find and interpret μ_M.

(c) Find and interpret σ_M.

38. **Skee Ball** Ana is a dedicated Skee Ball player (see photo) who always rolls for the 50-point slot. The probability distribution of Ana's score X on a single roll of the ball is shown below. You can check that $\mu_X = 23.8$ and $\sigma_X = 12.63$.

Score:	10	20	30	40	50
Probability:	0.32	0.27	0.19	0.15	0.07



(a) A player receives one ticket from the game for every 10 points scored. Make a graph of the probability distribution for the random variable T = number of tickets Ana gets on a randomly selected throw. Describe its shape.

(b) Find and interpret μ_T.

(c) Find and interpret σ_T.

Exercises 39 and 40 refer to the following setting. Ms. Hall gave her class a 10-question multiple-choice quiz. Let X = the number of questions that a randomly selected student in the class answered correctly. The computer output below gives information about the probability distribution of X. To determine each student's grade on the quiz (out of 100), Ms. Hall will multiply his or her number of correct answers by 5 and then add 50. Let G = the grade of a randomly chosen student in the class.

	Mean	Median	StDev	Min	Max	Q_1	Q_3
30	7.6	8.5	1.32	4	10	8	9

39. **Easy quiz**

(a) Find the mean of G. Show your method.

(b) Find the standard deviation of G. Show your method.

(c) How do the variance of X and the variance of G compare? Justify your answer.

40. **Easy quiz**

(a) Find the median of G. Show your method.

(b) Find the *IQR* of G. Show your method.

(c) What shape would the probability distribution of G have? Justify your answer.

41. **Get on the boat!** Refer to Exercise 37. The ferry company's expenses are \$20 per trip. Define the random variable Y to be the amount of profit (money collected minus expenses) made by the ferry company on a randomly selected trip. That is, $Y = M - 20$.

(a) Find and interpret the mean of Y.

(b) Find and interpret the standard deviation of Y.

42. **The Tri-State Pick 3** Most states and Canadian provinces have government-sponsored lotteries. Here is a simple lottery wager, from the Tri-State Pick 3 game that New Hampshire shares with Maine and Vermont. You choose a number with 3 digits from 0 to 9; the state chooses a three-digit winning number at random and pays you \$500 if your number is chosen. Because there are 1000 numbers with three digits, you have probability 1/1000 of winning. Taking X to be the amount your ticket pays you, the probability distribution of X is

Payoff:	\$0	\$500
Probability:	0.999	0.001

(a) Show that the mean and standard deviation of X are $\mu_X = \$0.50$ and $\sigma_X = \$15.80$.

(b) If you buy a Pick 3 ticket, your winnings are $W = X - 1$, because it costs \$1 to play. Find the mean and standard deviation of W. Interpret each of these values in context.

43. **Get on the boat!** Based on the analysis in Exercise 41, the ferry company decides to increase the cost of a trip to \$6. We can calculate the company's profit Y on a randomly selected trip from the number of cars X. Find the mean and standard deviation of Y. Show your work.

44. **Making a profit** Rotter Partners is planning a major investment. From experience, the amount of profit X (in millions of dollars) on a randomly selected investment of this type is uncertain, but an estimate gives the following probability distribution:

Profit:	1	1.5	2	4	10
Probability:	0.1	0.2	0.4	0.2	0.1

Based on this estimate, $\mu_X = 3$ and $\sigma_X = 2.52$. Rotter Partners owes its lender a fee of \$200,000 plus 10% of the profits X. So the firm actually retains $Y = 0.9X - 0.2$ from the investment. Find the mean and standard deviation of Y. Show your work.

45. **Too cool at the cabin?** During the winter months, the temperatures at the Starneses' Colorado cabin can stay well below freezing (32°F or 0°C) for weeks at a time. To prevent the pipes from freezing, Mrs. Starnes sets the thermostat at 50°F. She also buys a digital thermometer that records the indoor temperature each night at midnight. Unfortunately, the thermometer is programmed to measure the temperature in degrees Celsius. Based on several years' worth of data, the temperature T in the cabin at midnight on a randomly selected night follows a Normal distribution with mean 8.5°C and standard deviation 2.25°C.

pg 368

(a) Let Y = the temperature in the cabin at midnight on a randomly selected night in degrees Fahrenheit (recall that $F = (9/5)C + 32$). Find the mean and standard deviation of Y.

(b) Find the probability that the midnight temperature in the cabin is below 40°F. Show your work.

46. **Cereal** A company's single-serving cereal boxes advertise 9.63 ounces of cereal. In fact, the amount of cereal X in a randomly selected box follows a Normal distribution with a mean of 9.70 ounces and a standard deviation of 0.03 ounces.

(a) Let Y = the excess amount of cereal beyond what's advertised in a randomly selected box, measured in grams (1 ounce = 28.35 grams). Find the mean and standard deviation of Y.

(b) Find the probability of getting at least 3 grams more cereal than advertised. Show your work.

47. **His and her earnings** Researchers randomly select a married couple in which both spouses are employed. Let X be the income of the husband and Y be the income of the wife. Suppose that you know the means μ_X and μ_Y and the variances σ_X^2 and σ_Y^2 of both variables.

pg 373

(a) Is it reasonable to take the mean of the total income $X + Y$ to be $\mu_X + \mu_Y$? Explain your answer.

(b) Is it reasonable to take the variance of the total income to be $\sigma_X^2 + \sigma_Y^2$? Explain your answer.

48. **Rainy days** Imagine that we randomly select a day from the past 10 years. Let X be the recorded rainfall on this date at the airport in Orlando, Florida, and Y be the recorded rainfall on this date at Disney World just outside Orlando. Suppose that you know the means μ_X and μ_Y and the variances σ_X^2 and σ_Y^2 of both variables.

(a) Is it reasonable to take the mean of the total rainfall $X + Y$ to be $\mu_X + \mu_Y$? Explain your answer.

(b) Is it reasonable to take the variance of the total rainfall to be $\sigma_X^2 + \sigma_Y^2$? Explain your answer.

49. **Get on the boat!** Refer to Exercise 41. Find the expected value and standard deviation of the total amount of profit made on two randomly selected days. Show your work.

50. **The Tri-State Pick 3** Refer to Exercise 42. Suppose you buy one Pick 3 ticket on each of two consecutive days. Find the expected value and standard deviation of your total winnings. Show your work.

51. **Essay errors** Typographical and spelling errors can be either "nonword errors" or "word errors." A nonword error is not a real word, as when "the" is typed as "teh." A word error is a real word, but not the right word, as when "lose" is typed as "loose." When students are asked to write a 250-word essay (without spell-checking), the number of nonword errors X in a randomly selected essay has the following probability distribution:

pg 374

Value:	0	1	2	3	4
Probability:	0.1	0.2	0.3	0.3	0.1

$\mu_X = 2.1 \qquad \sigma_X = 1.136$

The number of word errors Y has this probability distribution:

Value:	0	1	2	3
Probability:	0.4	0.3	0.2	0.1

$\mu_Y = 1.0 \qquad \sigma_Y = 1.0$

Assume that X and Y are independent.

An English professor deducts 3 points from a student's essay score for each nonword error and 2 points for each word error. Find the mean and standard deviation of the total score deductions for a randomly selected essay. Show your work.

52. **The Tri-State Pick 3** Refer to Exercise 42. You and a friend decide to play Pick 3, but with two different strategies. Your friend buys a \$1 Pick 3 ticket on each of five consecutive days. You bet \$5 on a single number on your Pick 3 ticket. Find the mean and standard deviation of the total winnings for you and your friend. Show your work.

53. **Essay errors** Refer to Exercise 51.

(a) Find the mean and standard deviation of the difference $Y - X$ in the number of errors made by a randomly selected student. Interpret each value in context.

(b) From the information given, can you find the probability that a randomly selected student makes more word errors than nonword errors? If so, find this probability. If not, explain why not.

54. **Study habits** The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures academic motivation and study habits. The distribution of SSHA scores among the women at a college has mean 120 and standard deviation 28, and the distribution of scores among male students has mean

105 and standard deviation 35. You select a single male student and a single female student at random and give them the SSHA test.

(a) Find the mean and standard deviation of the difference (female minus male) between their scores. Interpret each value in context.

(b) From the information given, can you find the probability that the woman chosen scores higher than the man? If so, find this probability. If not, explain why you cannot.

55. **Essay scores** Refer to Exercise 51. Find the mean and standard deviation of the difference in score deductions (nonword – word) for a randomly selected essay. Show your work.

pg 377

56. **The Tri-State Pick 3** Refer to Exercise 52. Find the mean and standard deviation of the difference (you – your friend) in winnings. Show your work.

Exercises 57 and 58 refer to the following setting. In Exercises 14 and 18 of Section 6.1, we examined the probability distribution of the random variable X = the amount a life insurance company earns on a randomly chosen 5-year term life policy. Calculations reveal that $\mu_X = \$303.35$ and $\sigma_X = \$9707.57$.

57. **Life insurance** The risk of insuring one person's life is reduced if we insure many people. Suppose that we insure two 21-year-old males, and that their ages at death are independent. If X_1 and X_2 are the insurer's income from the two insurance policies, the insurer's average income W on the two policies is

$$W = \frac{X_1 + X_2}{2}$$

Find the mean and standard deviation of W. (You see that the mean income is the same as for a single policy but the standard deviation is less.)

58. **Life insurance** If four 21-year-old men are insured, the insurer's average income is

$$V = \frac{X_1 + X_2 + X_3 + X_4}{4}$$

where X_i is the income from insuring one man. Assuming that the amount of income earned on individual policies is independent, find the mean and standard deviation of V. (If you compare with the results of Exercise 57, you should see that averaging over more insured individuals reduces risk.)

59. **Time and motion** A time-and-motion study measures the time required for an assembly-line worker to perform a repetitive task. The data show that the time required to bring a part from a bin to its position on an automobile chassis varies from car to car according to a Normal distribution with mean 11 seconds and standard deviation 2 seconds. The time required to attach the part to the chassis follows a Normal distribution with mean 20 seconds and standard deviation 4 seconds. The study finds that the times required for the two steps are independent. A part that takes a long time to position, for example, does not take more or less time to attach than other parts.

(a) What is the distribution of the time required for the entire operation of positioning and attaching a randomly selected part?

(b) Management's goal is for the entire process to take less than 30 seconds. Find the probability that this goal will be met for a randomly selected part.

60. **Electronic circuit** The design of an electronic circuit for a toaster calls for a 100-ohm resistor and a 250-ohm resistor connected in series so that their resistances add. The components used are not perfectly uniform, so that the actual resistances vary independently according to Normal distributions. The resistance of 100-ohm resistors has mean 100 ohms and standard deviation 2.5 ohms, while that of 250-ohm resistors has mean 250 ohms and standard deviation 2.8 ohms.

(a) What is the distribution of the total resistance of the two components in series for a randomly selected toaster?

(b) Find the probability that the total resistance for a randomly selected toaster lies between 345 and 355 ohms.

61. **Swim team** Hanover High School has the best women's swimming team in the region. The 400-meter freestyle relay team is undefeated this year. In the 400-meter freestyle relay, each swimmer swims 100 meters. The times, in seconds, for the four swimmers this season are approximately Normally distributed with means and standard deviations as shown. Assume that the swimmer's individual times are independent. Find the probability that the total team time in the 400-meter freestyle relay for a randomly selected race is less than 220 seconds.

pg 380

Swimmer	Mean	Std. dev.
Wendy	55.2	2.8
Jill	58.0	3.0
Carmen	56.3	2.6
Latrice	54.7	2.7

62. **Toothpaste** Ken is traveling for his business. He has a new 0.85-ounce tube of toothpaste that's supposed to last him the whole trip. The amount of toothpaste Ken squeezes out of the tube each time he brushes varies according to a Normal distribution with mean 0.13 ounces and standard deviation 0.02 ounces. If Ken brushes his teeth six times on a randomly

63. **Auto emissions** The amount of nitrogen oxides (NOX) present in the exhaust of a particular type of car varies from car to car according to a Normal distribution with mean 1.4 grams per mile (g/mi) and standard deviation 0.3 g/mi. Two randomly selected cars of this type are tested. One has 1.1 g/mi of NOX; the other has 1.9 g/mi. The test station attendant finds this difference in emissions between two similar cars surprising. If the NOX levels for two randomly chosen cars of this type are independent, find the probability that the difference is at least as large as the value the attendant observed.

pg 381

64. **Loser buys the pizza** Leona and Fred are friendly competitors in high school. Both are about to take the ACT college entrance examination. They agree that if one of them scores 5 or more points better than the other, the loser will buy the winner a pizza. Suppose that in fact Fred and Leona have equal ability, so that each score varies Normally with mean 24 and standard deviation 2. (The variation is due to luck in guessing and the accident of the specific questions being familiar to the student.) The two scores are independent. What is the probability that the scores differ by 5 or more points in either direction?

Multiple choice: Select the best answer for Exercises 65 and 66, which refer to the following setting. The number of calories in a 1-ounce serving of a certain breakfast cereal is a random variable with mean 110 and standard deviation 10. The number of calories in a cup of whole milk is a random variable with mean 140 and standard deviation 12. For breakfast, you eat 1 ounce of the cereal with 1/2 cup of whole milk. Let T be the random variable that represents the total number of calories in this breakfast.

65. The mean of T is

(a) 110. (b) 140. (c) 180. (d) 195. (e) 250.

66. The standard deviation of T is

(a) 22. (b) 16. (c) 15.62. (d) 11.66. (e) 4.

67. **Statistics for investing** (3.1) Joe's retirement plan invests in stocks through an "index fund" that follows the behavior of the stock market as a whole, as measured by the Standard & Poor's (S&P) 500 stock index. Joe wants to buy a mutual fund that does not track the index closely. He reads that monthly returns from Fidelity Technology Fund have correlation $r = 0.77$ with the S&P 500 index and that Fidelity Real Estate Fund has correlation $r = 0.37$ with the index.

(a) Which of these funds has the closer relationship to returns from the stock market as a whole? How do you know?

(b) Does the information given tell Joe anything about which fund has had higher returns?

68. **Buying stock** (5.3, 6.1) You purchase a hot stock for \$1000. The stock either gains 30% or loses 25% each day, each with probability 0.5. Its returns on consecutive days are independent of each other. You plan to sell the stock after two days.

(a) What are the possible values of the stock after two days, and what is the probability for each value? What is the probability that the stock is worth more after two days than the \$1000 you paid for it?

(b) What is the mean value of the stock after two days? (*Comment:* You see that these two criteria give different answers to the question "Should I invest?")

6.3 Binomial and Geometric Random Variables

WHAT YOU WILL LEARN By the end of the section, you should be able to:

- Determine whether the conditions for using a binomial random variable are met.
- Compute and interpret probabilities involving binomial distributions.
- Calculate the mean and standard deviation of a binomial random variable. Interpret these values in context.
- Find probabilities involving geometric random variables.
- When appropriate, use the Normal approximation to the binomial distribution to calculate probabilities.*

*This topic is not required for the AP® Statistics exam. Some teachers prefer to discuss this topic when presenting the sampling distribution of $\hat{p}$ (Chapter 7).