- The **general multiplication rule** states that the probability of events A and B occurring together is

$$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B \mid A)$$

- When chance behavior involves a sequence of outcomes, a **tree diagram** can be used to describe the sample space. Tree diagrams can also help in finding the probability that two or more events occur together. We simply multiply along the branches that correspond to the outcomes of interest.
- When knowing that one event has occurred does not change the probability that another event happens, we say that the two events are **independent**. For independent events A and B, $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$. If two events A and B are mutually exclusive (disjoint), they cannot be independent.
- In the special case of *independent* events, the multiplication rule becomes

$$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B)$$

Section 5.3 Exercises

63. **Get rich** A survey of 4826 randomly selected young adults (aged 19 to 25) asked, "What do you think are the chances you will have much more than a middle-class income at age 30?" The two-way table shows the responses.[16] Choose a survey respondent at random.

	Gender		
Opinion	**Female**	**Male**	**Total**
Almost no chance	96	98	**194**
Some chance but probably not	426	286	**712**
A 50-50 chance	696	720	**1416**
A good chance	663	758	**1421**
Almost certain	486	597	**1083**
Total	**2367**	**2459**	**4826**

(a) Given that the person selected is male, what's the probability that he answered "almost certain"?

(b) If the person selected said "some chance but probably not," what's the probability that the person is female?

64. **A *Titanic* disaster** In 1912 the luxury liner *Titanic*, on its first voyage across the Atlantic, struck an iceberg and sank. Some passengers got off the ship in lifeboats, but many died. The two-way table gives information about adult passengers who lived and who died, by class of travel. Suppose we choose an adult passenger at random.

	Survival Status	
Class of Travel	Survived	Died
First class	197	122
Second class	94	167
Third class	151	476

(a) Given that the person selected was in first class, what's the probability that he or she survived?

(b) If the person selected survived, what's the probability that he or she was a third-class passenger?

65. **Sampling senators** The two-way table describes the members of the U.S. Senate in a recent year. Suppose we select a senator at random. Consider events D: is a democrat, and F: is female.

	Male	Female
Democrats	47	13
Republicans	36	4

(a) Find $P(D \mid F)$. Explain what this value means.

(b) Find $P(F \mid D)$. Explain what this value means.

66. **Who eats breakfast?** The following two-way table describes the 595 students who responded to a school survey about eating breakfast. Suppose we select a student at random. Consider events B: eats breakfast regularly, and M: is male.

	Male	Female	Total
Eats breakfast regularly	190	110	300
Doesn't eat breakfast regularly	130	165	295
Total	320	275	595

(a) Find $P(B \mid M)$. Explain what this value means.

(b) Find $P(M \mid B)$. Explain what this value means.

67. **Foreign-language study** Choose a student in grades 9 to 12 at random and ask if he or she is studying a language other than English. Here is the distribution of results:

Language:	Spanish	French	German	All others	None
Probability:	0.26	0.09	0.03	0.03	0.59

(a) What's the probability that the student is studying a language other than English?

(b) What is the conditional probability that a student is studying Spanish given that he or she is studying some language other than English?

68. **Income tax returns** Here is the distribution of the adjusted gross income (in thousands of dollars) reported on individual federal income tax returns in a recent year:

Income:	<25	25–49	50–99	100–499	≥500
Probability:	0.431	0.248	0.215	0.100	0.006

(a) What is the probability that a randomly chosen return shows an adjusted gross income of $50,000 or more?

(b) Given that a return shows an income of at least $50,000, what is the conditional probability that the income is at least $100,000?

69. **Tall people and basketball players** Select an adult at random. Define events T: person is over 6 feet tall, and B: person is a professional basketball player. Rank the following probabilities from smallest to largest. Justify your answer.

$P(T)$ $P(B)$ $P(T \mid B)$ $P(B \mid T)$

70. **Teachers and college degrees** Select an adult at random. Define events A: person has earned a college degree, and T: person's career is teaching. Rank the following probabilities from smallest to largest. Justify your answer.

$P(A)$ $P(T)$ $P(A \mid T)$ $P(T \mid A)$

71. **Facebook versus YouTube** A recent survey suggests that 85% of college students have posted a profile on Facebook, 73% use YouTube regularly, and 66% do both. Suppose we select a college student at random and learn that the student has a profile on Facebook. Find the probability that the student uses YouTube regularly. Show your work.

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72. **Mac or PC?** A recent census at a major university revealed that 40% of its students mainly used Macintosh computers (Macs). The rest mainly used PCs. At the time of the census, 67% of the school's students were undergraduates. The rest were graduate students. In the census, 23% of the respondents were graduate students who said that they used PCs as their primary computers. Suppose we select a student at random from among those who were part of the census and learn that the student mainly uses a PC. Find the probability that this person is a graduate student. Show your work.

73. **Free downloads?** Illegal music downloading has become a big problem: 29% of Internet users download music files, and 67% of downloaders say they don't care if the music is copyrighted.[17] What percent of Internet users download music and don't care if it's copyrighted? Write the information given in terms of probabilities, and use the general multiplication rule.

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74. **At the gym** Suppose that 10% of adults belong to health clubs, and 40% of these health club members go to the club at least twice a week. What percent of all adults go to a health club at least twice a week? Write the information given in terms of probabilities, and use the general multiplication rule.

75. **Box of chocolates** According to Forrest Gump, "Life is like a box of chocolates. You never know what you're gonna get." Suppose a candy maker offers a special "Gump box" with 20 chocolate candies that look the same. In fact, 14 of the candies have soft centers and 6 have hard centers. Choose 2 of the candies from a Gump box at random.

(a) Draw a tree diagram that shows the sample space of this chance process.

(b) Find the probability that one of the chocolates has a soft center and the other one doesn't.

76. **Inspecting switches** A shipment contains 10,000 switches. Of these, 1000 are bad. An inspector draws 2 switches at random, one after the other.

(a) Draw a tree diagram that shows the sample space of this chance process.

(b) Find the probability that both switches are defective.

77. **Fill 'er up!** In a recent month, 88% of automobile drivers filled their vehicles with regular gasoline, 2% purchased midgrade gas, and 10% bought premium gas.[18] Of those who bought regular gas, 28% paid with a credit card; of customers who bought midgrade and premium gas, 34% and 42%, respectively, paid with a credit card. Suppose we select a customer at random.

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(a) Draw a tree diagram to represent this situation.

(b) Find the probability that the customer paid with a credit card. Show your work.

(c) Given that the customer paid with a credit card, find the probability that she bought premium gas. Show your work.

78. **Urban voters** The voters in a large city are 40% white, 40% black, and 20% Hispanic. (Hispanics may be of any race in official statistics, but here we are speaking of political blocks.) A mayoral candidate anticipates attracting 30% of the white vote, 90% of the black vote, and 50% of the Hispanic vote. Suppose we select a voter at random.

(a) Draw a tree diagram to represent this situation.

(b) Find the probability that this voter votes for the mayoral candidate. Show your work.

(c) Given that the chosen voter plans to vote for the candidate, find the probability that the voter is black. Show your work.

79. **Lactose intolerance** Lactose intolerance causes difficulty in digesting dairy products that contain lactose (milk sugar). It is particularly common among people of African and Asian ancestry. In the United States (ignoring other groups and people who consider themselves to belong to more than one race), 82% of the population is white, 14% is black, and 4% is Asian. Moreover, 15% of whites, 70% of blacks, and 90% of Asians are lactose intolerant.[19] Suppose we select a U.S. person at random.

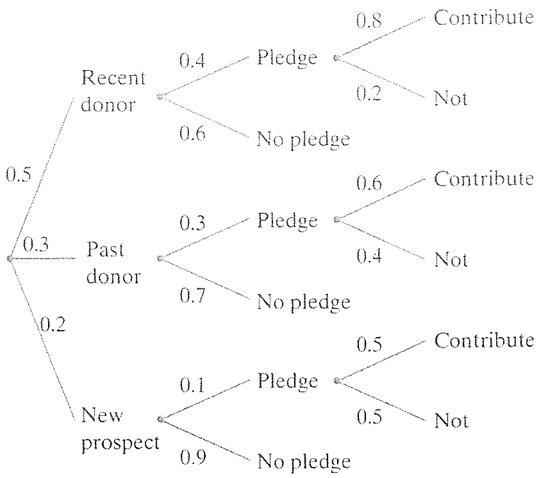
(a) What is the probability that the person is lactose intolerant? Show your work.

(b) Given that the person is lactose intolerant, find the probability that he or she is Asian. Show your work.

80. **Fundraising by telephone** Tree diagrams can organize problems having more than two stages. The figure at top right shows probabilities for a charity calling potential donors by telephone.[20] Each person called is either a recent donor, a past donor, or a new prospect. At the next stage, the person called either does or does not pledge to contribute, with conditional probabilities that depend on the donor class to which the person belongs. Finally, those who make a pledge either do or don't actually make a contribution. Suppose we randomly select a person who is called by the charity.

(a) What is the probability that the person contributed to the charity? Show your work.

(b) Given that the person contributed, find the probability that he or she is a recent donor. Show your work.



81. **HIV testing** Enzyme immunoassay (EIA) tests are used to screen blood specimens for the presence of antibodies to HIV, the virus that causes AIDS. Antibodies indicate the presence of the virus. The test is quite accurate but is not always correct. Here are approximate probabilities of positive and negative EIA outcomes when the blood tested does and does not actually contain antibodies to HIV:[21]

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	Test Result	
Truth	+	−
Antibodies present	0.9985	0.0015
Antibodies absent	0.006	0.994

Suppose that 1% of a large population carries antibodies to HIV in their blood. We choose a person from this population at random. Given that the EIA test is positive, find the probability that the person has the antibody. Show your work.

82. **Testing the test** Are false positives too common in some medical tests? Researchers conducted an experiment involving 250 patients with a medical condition and 750 other patients who did not have the medical condition. The medical technicians who were reading the test results were unaware that they were subjects in an experiment.

(a) Technicians correctly identified 240 of the 250 patients with the condition. They also identified 50 of the healthy patients as having the condition. What were the false positive and false negative rates for the test?

(b) Given that a patient got a positive test result, what is the probability that the patient actually had the medical condition? Show your work.

83. **Get rich** Refer to Exercise 63.

(a) Find P("a good chance" | female).

(b) Find P("a good chance").

(c) Use your answers to (a) and (b) to determine whether the events "a good chance" and "female" are independent. Explain your reasoning.

84. **A *Titanic* disaster** Refer to Exercise 64.

(a) Find P(survived | second class).

(b) Find P(survived).

(c) Use your answers to (a) and (b) to determine whether the events "survived" and "second class" are independent. Explain your reasoning.

pg 327 85. **Sampling senators** Refer to Exercise 65. Are events D and F independent? Justify your answer.

86. **Who eats breakfast?** Refer to Exercise 66. Are events B and M independent? Justify your answer.

87. **Rolling dice** Suppose you roll two fair, six-sided dice—one red and one green. Are the events "sum is 7" and "green die shows a 4" independent? Justify your answer.

88. **Rolling dice** Suppose you roll two fair, six-sided dice—one red and one green. Are the events "sum is 8" and "green die shows a 4" independent? Justify your answer.

89. **Bright lights?** pg 329 A string of Christmas lights contains 20 lights. The lights are wired in series, so that if any light fails, the whole string will go dark. Each light has probability 0.02 of failing during a 3-year period. The lights fail independently of each other. Find the probability that the string of lights will remain bright for 3 years.

90. **Common names** The Census Bureau says that the 10 most common names in the United States are (in order) Smith, Johnson, Williams, Brown, Jones, Miller, Davis, Garcia, Rodriguez, and Wilson. These names account for 9.6% of all U.S. residents. Out of curiosity, you look at the authors of the textbooks for your current courses. There are 9 authors in all. Would you be surprised if none of the names of these authors were among the 10 most common? (Assume that authors' names are independent and follow the same probability distribution as the names of all residents.)

91. **Universal blood donors** pg 329 People with type O-negative blood are universal donors. That is, any patient can receive a transfusion of O-negative blood. Only 7.2% of the American population have O-negative blood. If we choose 10 Americans at random who gave blood, what is the probability that at least 1 of them is a universal donor?

92. **Lost Internet sites** Internet sites often vanish or move, so that references to them can't be followed. In fact, 13% of Internet sites referenced in major scientific journals are lost within two years after publication.[22] If we randomly select seven Internet references, from scientific journals, what is the probability that at least one of them doesn't work two years later?

93. **Late shows** Some TV shows begin after their scheduled times when earlier programs run late. According to a network's records, about 3% of its shows start late. To find the probability that three consecutive shows on this network start on time, can we multiply (0.97)(0.97)(0.97)? Why or why not?

94. **Late flights** An airline reports that 85% of its flights arrive on time. To find the probability that its next four flights into LaGuardia Airport all arrive on time, can we multiply (0.85)(0.85)(0.85)(0.85)? Why or why not?

95. **The geometric distributions** You are tossing a pair of fair, six-sided dice in a board game. Tosses are independent. You land in a danger zone that requires you to roll doubles (both faces showing the same number of spots) before you are allowed to play again. How long will you wait to play again?

(a) What is the probability of rolling doubles on a single toss of the dice? (If you need review, the possible outcomes appear in Figure 5.2 (page 306). All 36 outcomes are equally likely.)

(b) What is the probability that you do not roll doubles on the first toss, but you do on the second toss?

(c) What is the probability that the first two tosses are not doubles and the third toss is doubles? This is the probability that the first doubles occurs on the third toss.

(d) Now you see the pattern. What is the probability that the first doubles occurs on the fourth toss? On the fifth toss? Give the general result: what is the probability that the first doubles occurs on the kth toss?

96. **The probability of a flush** A poker player holds a flush when all 5 cards in the hand belong to the same suit. We will find the probability of a flush when 5 cards are dealt. Remember that a deck contains 52 cards, 13 of each suit, and that when the deck is well shuffled, each card dealt is equally likely to be any of those that remain in the deck.

(a) We will concentrate on spades. What is the probability that the first card dealt is a spade? What is the conditional probability that the second card is a spade given that the first is a spade?

(b) Continue to count the remaining cards to find the conditional probabilities of a spade on the third, the fourth, and the fifth card given in each case that all previous cards are spades.

(c) The probability of being dealt 5 spades is the product of the five probabilities you have found. Why? What is this probability?

(d) The probability of being dealt 5 hearts or 5 diamonds or 5 clubs is the same as the probability of being dealt 5 spades. What is the probability of being dealt a flush?

Multiple choice: Select the best answer for Exercises 97 to 99.

97. An athlete suspected of using steroids is given two tests that operate independently of each other. Test A has probability 0.9 of being positive if steroids have been used. Test B has probability 0.8 of being positive if steroids have been used. What is the probability that neither test is positive if steroids have been used?

(a) 0.72 (c) 0.02 (e) 0.08

(b) 0.38 (d) 0.28

98. In an effort to find the source of an outbreak of food poisoning at a conference, a team of medical detectives carried out a study. They examined all 50 people who had food poisoning and a random sample of 200 people attending the conference who didn't get food poisoning. The detectives found that 40% of the people with food poisoning went to a cocktail party on the second night of the conference, while only 10% of the people in the random sample attended the same party. Which of the following statements is appropriate for describing the 40% of people who went to the party? (Let F = got food poisoning and A = attended party.)

(a) $P(F \mid A) = 0.40$ (d) $P(A^C \mid F) = 0.40$

(b) $P(A \mid F^C) = 0.40$ (e) $P(A \mid F) = 0.40$

(c) $P(F \mid A^C) = 0.40$

99. Suppose a loaded die has the following probability model:

Outcome:	1	2	3	4	5	6
Probability:	0.3	0.1	0.1	0.1	0.1	0.3

If this die is thrown and the top face shows an odd number, what is the probability that the die shows a 1?

(a) 0.10 (d) 0.50

(b) 0.17 (e) 0.60

(c) 0.30

Exercises 100 and 101 refer to the following setting. Your body mass index (BMI) is your weight in kilograms divided by the square of your height in meters. Online BMI calculators allow you to enter weight in pounds and height in inches. High BMI is a common but controversial indicator of being overweight or obese. A study by the National Center for Health Statistics found that the BMI of American young women (ages 20 to 29) is approximately Normal with mean 26.8 and standard deviation 7.4.[23]

100. **BMI** (2.2) People with BMI less than 18.5 are often classed as "underweight." What percent of young women are underweight by this criterion? Sketch and shade the area of interest under a Normal curve.

101. **BMI** (5.2) Suppose we select two American young women in this age group at random. Find the probability that at least one of them is classified as underweight. Show your work.

102. **Life at work** (1.1) The University of Chicago's General Social Survey asked a representative sample of adults this question: "Which of the following statements best describes how your daily work is organized? (1) I am free to decide how my daily work is organized. (2) I can decide how my daily work is organized, within certain limits. (3) I am not free to decide how my daily work is organized." Here is a two-way table of the responses for three levels of education:[24]

	Highest Degree Completed		
Response	Less than High School	High School	Bachelor's
1	31	161	81
2	49	269	85
3	47	112	14

Do these data suggest that there is an association between level of education and freedom to organize one's work in the adult population? Give appropriate evidence to support your answer.