In the previous example, the event "reads neither paper" is the complement of the event "reads at least one of the papers." To solve part (d) of the problem, we could have used our answer from (c) and the complement rule:

$$P(\text{reads neither paper}) = 1 - P(\text{reads at least one paper}) = 1 - 0.60 = 0.40$$

As you'll see in Section 5.3, the fact that "none" is the opposite of "at least one" comes in handy for a variety of probability questions.

# Section 5.2 Summary

- A **probability model** describes chance behavior by listing the possible outcomes in the **sample space** $S$ and giving the probability that each outcome occurs.
- An **event** is a subset of the possible outcomes in the sample space. To find the probability that an event $A$ happens, we can rely on some basic probability rules:
  - For any event $A$, $0 \le P(A) \le 1$.
  - $P(S) = 1$, where $S$ = the sample space
  - If all outcomes in the sample space are equally likely,

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

  - *Complement rule:* $P(A^C) = 1 - P(A)$, where $A^C$ is the **complement** of event $A$; that is, the event that $A$ does not happen.
  - *Addition rule for mutually exclusive events:* Events $A$ and $B$ are **mutually exclusive (disjoint)** if they have no outcomes in common. If $A$ and $B$ are disjoint, $P(A \text{ or } B) = P(A) + P(B)$.
- A **two-way table** or a **Venn diagram** can be used to display the sample space for a chance process. Two-way tables and Venn diagrams can also be used to find probabilities involving events $A$ and $B$, like the **union** ($A \cup B$) and **intersection** ($A \cap B$). The event $A \cup B$ ("$A$ or $B$") consists of all outcomes in event $A$, event $B$, or both. The event $A \cap B$ ("$A$ and $B$") consists of outcomes in both $A$ and $B$.
- The **general addition rule** can be used to find $P(A \text{ or } B)$:

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

# Section 5.2 Exercises

39. **Role-playing games** Computer games in which the players take the roles of characters are very popular. They go back to earlier tabletop games such as Dungeons & Dragons. These games use many different types of dice. A four-sided die has faces with 1, 2, 3, and 4 spots.

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(a) List the sample space for rolling the die twice (spots showing on first and second rolls).

(b) What is the assignment of probabilities to outcomes in this sample space? Assume that the die is perfectly balanced.

40. **Tossing coins** Imagine tossing a fair coin 3 times.

(a) What is the sample space for this chance process?

(b) What is the assignment of probabilities to outcomes in this sample space?

41. **Role-playing games** Refer to Exercise 39. Define event $A$: sum is 5. Find $P(A)$.

42. **Tossing coins** Refer to Exercise 40. Define event $B$: get more heads than tails. Find $P(B)$.

43. **Probability models?** In each of the following situations, state whether or not the given assignment of probabilities to individual outcomes is legitimate, that is, satisfies the rules of probability. If not, give specific reasons for your answer.

(a) Roll a 6-sided die and record the count of spots on the up-face: $P(1) = 0$, $P(2) = 1/6$, $P(3) = 1/3$, $P(4) = 1/3$, $P(5) = 1/6$, $P(6) = 0$.

(b) Choose a college student at random and record gender and enrollment status: $P(\text{female full-time}) = 0.56$, $P(\text{male full-time}) = 0.44$, $P(\text{female part-time}) = 0.24$, $P(\text{male part-time}) = 0.17$.

(c) Deal a card from a shuffled deck: $P(\text{clubs}) = 12/52$, $P(\text{diamonds}) = 12/52$, $P(\text{hearts}) = 12/52$, $P(\text{spades}) = 16/52$.

44. **Rolling a die** The following figure displays several possible probability models for rolling a die. Some of the models are not *legitimate*. That is, they do not obey the rules. Which are legitimate and which are not? In the case of the illegitimate models, explain what is wrong.

| | Probability | | | |
|---|---|---|---|---|
| **Outcome** | Model 1 | Model 2 | Model 3 | Model 4 |
|  | 1/7 | 1/3 | 1/3 | 1 |
|  | 1/7 | 1/6 | 1/6 | 1 |
|  | 1/7 | 1/6 | 1/6 | 2 |
|  | 1/7 | 0 | 1/6 | 1 |
|  | 1/7 | 1/6 | 1/6 | 1 |
|  | 1/7 | 1/6 | 1/6 | 2 |

45. **Blood types** All human blood can be typed as one of O, A, B, or AB, but the distribution of the types varies a bit with race. Here is the distribution of the blood type of a randomly chosen black American:

| **Blood type:** | O | A | B | AB |
|---|---|---|---|---|
| **Probability:** | 0.49 | 0.27 | 0.20 | ? |

(a) What is the probability of type AB blood? Why?

(b) What is the probability that the person chosen does not have type AB blood?

(c) Maria has type B blood. She can safely receive blood transfusions from people with blood types O and B. What is the probability that a randomly chosen black American can donate blood to Maria?

46. **Languages in Canada** Canada has two official languages, English and French. Choose a Canadian at random and ask, "What is your mother tongue?" Here is the distribution of responses, combining many separate languages from the broad Asia/Pacific region:[7]

| **Language:** | English | French | Asian/Pacific | Other |
|---|---|---|---|---|
| **Probability:** | 0.63 | 0.22 | 0.06 | ? |

(a) What probability should replace "?" in the distribution? Why?

(b) What is the probability that a Canadian's mother tongue is not English?

(c) What is the probability that a Canadian's mother tongue is a language other than English or French?

47. **Education among young adults** Choose a young adult (aged 25 to 29) at random. The probability is 0.13 that the person chosen did not complete high school, 0.29 that the person has a high school diploma but no further education, and 0.30 that the person has at least a bachelor's degree.

(a) What must be the probability that a randomly chosen young adult has some education beyond high school but does not have a bachelor's degree? Why?

(b) What is the probability that a randomly chosen young adult has at least a high school education? Which rule of probability did you use to find the answer?

48. **Preparing for the GMAT** A company that offers courses to prepare students for the Graduate Management Admission Test (GMAT) has the following information about its customers: 20% are currently undergraduate students in business; 15% are undergraduate students in other fields of study; 60% are college graduates who are currently employed; and 5% are college graduates who are not employed. Choose a customer at random.

(a) What's the probability that the customer is currently an undergraduate? Which rule of probability did you use to find the answer?

(b) What's the probability that the customer is not an undergraduate business student? Which rule of probability did you use to find the answer?

49. **Who eats breakfast?** Students in an urban school were curious about how many children regularly eat breakfast. They conducted a survey, asking, "Do you

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eat breakfast on a regular basis?" All 595 students in the school responded to the survey. The resulting data are shown in the two-way table below.[8]

| | Male | Female | Total |
|---|---|---|---|
| Eats breakfast regularly | 190 | 110 | 300 |
| Doesn't eat breakfast regularly | 130 | 165 | 295 |
| **Total** | **320** | **275** | **595** |

If we select a student from the school at random, what is the probability that the student is

(a) a female?

(b) someone who eats breakfast regularly?

(c) a female and eats breakfast regularly?

(d) a female or eats breakfast regularly?

50. **Sampling senators** The two-way table below describes the members of the U.S Senate in a recent year.

| | Male | Female |
|---|---|---|
| Democrats | 47 | 13 |
| Republicans | 36 | 4 |

If we select a U.S. senator at random, what's the probability that the senator is

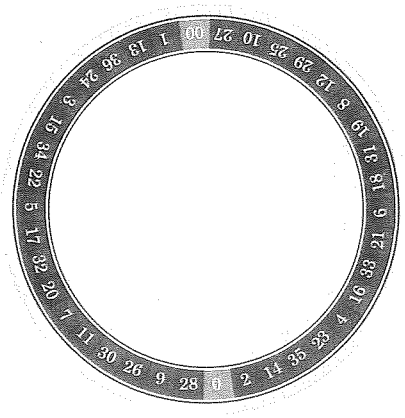
(a) a Democrat?

(b) a female?

(c) a female and a Democrat?

(d) a female or a Democrat?

51. **Roulette** An American roulette wheel has 38 slots with numbers 1 through 36, 0, and 00, as shown in the figure. Of the numbered slots, 18 are red, 18 are black, and 2—the 0 and 00—are green. When the wheel is spun, a metal ball is dropped onto the middle of the wheel. If the wheel is balanced, the ball is equally likely to settle in any of the numbered slots. Imagine spinning a fair wheel once. Define events B: ball lands in a black slot, and E: ball lands in an even-numbered slot. (Treat 0 and 00 as even numbers.)




(a) Make a two-way table that displays the sample space in terms of events B and E.

(b) Find $P(B)$ and $P(E)$.

(c) Describe the event "B and E" in words. Then find $P(B \text{ and } E)$.

(d) Explain why $P(B \text{ or } E) \neq P(B) + P(E)$. Then use the general addition rule to compute $P(B \text{ or } E)$.

52. **Playing cards** Shuffle a standard deck of playing cards and deal one card. Define events J: getting a jack, and R: getting a red card.

(a) Construct a two-way table that describes the sample space in terms of events J and R.

(b) Find $P(J)$ and $P(R)$.

(c) Describe the event "J and R" in words. Then find $P(J \text{ and } R)$.

(d) Explain why $P(J \text{ or } R) \neq P(J) + P(R)$. Then use the general addition rule to compute $P(J \text{ or } R)$.

53. **Who eats breakfast?** Refer to Exercise 49.

(a) Construct a Venn diagram that models the chance process using events B: eats breakfast regularly, and M: is male.

(b) Find $P(B \cup M)$. Interpret this value in context.

(c) Find $P(B^C \cap M^C)$. Interpret this value in context.

54. **Sampling senators** Refer to Exercise 50.

(a) Construct a Venn diagram that models the chance process using events R: is a Republican, and F: is female.

(b) Find $P(R \cup F)$. Interpret this value in context.

(c) Find $P(R^C \cap F^C)$. Interpret this value in context.

55. **Facebook versus YouTube** A recent survey suggests that 85% of college students have posted a profile on Facebook, 73% use YouTube regularly, and 66% do both. Suppose we select a college student at random.

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(a) Make a two-way table for this chance process.

(b) Construct a Venn diagram to represent this setting.

(c) Consider the event that the randomly selected college student has posted a profile on Facebook or uses YouTube regularly. Write this event in symbolic form based on your Venn diagram in part (b).

(d) Find the probability of the event described in part (c). Explain your method.

56. **Mac or PC?** A recent census at a major university revealed that 40% of its students mainly used Macintosh computers (Macs). The rest mainly used PCs. At the time of the census, 67% of the school's students were undergraduates. The rest were graduate students.

In the census, 23% of respondents were graduate students who said that they used PCs as their main computers. Suppose we select a student at random from among those who were part of the census.

(a) Make a two-way table for this chance process.

(b) Construct a Venn diagram to represent this setting.

(c) Consider the event that the randomly selected student is a graduate student and uses a Mac. Write this event in symbolic form based on your Venn diagram in part (b).

(d) Find the probability of the event described in part (c). Explain your method.

*Multiple choice: Select the best answer for Exercises 57 to 60.*

57. In government data, a household consists of all occupants of a dwelling unit. Choose an American household at random and count the number of people it contains. Here is the assignment of probabilities for the outcome:

| Number of persons: | 1 | 2 | 3 | 4 | 5 | 6 | 7+ |
|---|---|---|---|---|---|---|---|
| Probability: | 0.25 | 0.32 | ??? | ??? | 0.07 | 0.03 | 0.01 |

The probability of finding 3 people in a household is the same as the probability of finding 4 people. These probabilities are marked ??? in the table of the distribution. The probability that a household contains 3 people must be

(a) 0.68. (b) 0.32. (c) 0.16. (d) 0.08.

(e) between 0 and 1, and we can say no more.

58. In a sample of 275 students, 20 say they are vegetarians. Of the vegetarians, 9 eat both fish and eggs, 3 eat eggs but not fish, and 7 eat neither. Choose one of the vegetarians at random. What is the probability that the chosen student eats fish or eggs?

(a) 9/20 (c) 22/20 (e) 22/275

(b) 13/20 (d) 9/275

*Exercises 59 and 60 refer to the following setting.* The casino game craps is based on rolling two dice. Here is the assignment of probabilities to the sum of the numbers on the up-faces when two dice are rolled:

| Outcome: | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Probability: | 1/36 | 2/36 | 3/36 | 4/36 | 5/36 | 6/36 | 5/36 | 4/36 | 3/36 | 2/36 | 1/36 |

59. The most common bet in craps is the "pass line." A pass line bettor wins immediately if either a 7 or an 11 comes up on the first roll. This is called a *natural*. What is the probability of a natural?

(a) 2/36 (c) 8/36 (e) 20/36

(b) 6/36 (d) 12/36

60. If a player rolls a 2, 3, or 12, it is called *craps*. What is the probability of getting craps or an even sum on one roll of the dice?

(a) 4/36 (c) 20/36 (e) 32/36

(b) 18/36 (d) 22/36

61. **Crawl before you walk** (3.2) At what age do babies learn to crawl? Does it take longer to learn in the winter, when babies are often bundled in clothes that restrict their movement? Perhaps there might even be an association between babies' crawling age and the average temperature during the month they first try to crawl (around six months after birth). Data were collected from parents who brought their babies to the University of Denver Infant Study Center to participate in one of a number of studies. Parents reported the birth month and the age at which their child was first able to creep or crawl a distance of 4 feet within one minute. Information was obtained on 414 infants (208 boys and 206 girls). Crawling age is given in weeks, and average temperature (in °F) is given for the month that is six months after the birth month.[9]

| Birth month | Average crawling age | Average temperature |
|---|---|---|
| January | 29.84 | 66 |
| February | 30.52 | 73 |
| March | 29.70 | 72 |
| April | 31.84 | 63 |
| May | 28.58 | 52 |
| June | 31.44 | 39 |
| July | 33.64 | 33 |
| August | 32.82 | 30 |
| September | 33.83 | 33 |
| October | 33.35 | 37 |
| November | 33.38 | 48 |
| December | 32.32 | 57 |

Analyze the relationship between average crawling age and average temperature. What do you conclude about when babies learn to crawl?

62. **Treating low bone density** (4.2) Fractures of the spine are common and serious among women with advanced osteoporosis (low mineral density in the bones). Can taking strontium ranelate help? A large medical trial assigned 1649 women to take either strontium ranelate or a placebo each day. All of the subjects had osteoporosis and had had at least one fracture. All were taking calcium supplements and receiving standard medical care. The response variables were measurements of bone density and counts of new fractures over three years. The subjects were treated at 10 medical centers in 10 different countries.[10] Outline an appropriate design for this experiment. Explain why this is the proper design.