

The variable  $z$  has the **standard Normal distribution** with mean 0 and standard deviation 1.

- **Table A** at the back of the book gives percentiles for the standard Normal curve. By standardizing, we can use Table A to determine the percentile for a given  $z$ -score or the  $z$ -score corresponding to a given percentile in any Normal distribution. You can use your calculator or the *Normal Curve* applet to perform Normal calculations quickly.
- To perform certain inference procedures in later chapters, we will need to know that the data come from populations that are approximately Normally distributed. To assess Normality for a given set of data, we first observe the shape of a dotplot, stemplot, or histogram. Then we can check how well the data fit the 68–95–99.7 rule for Normal distributions. Another good method for assessing Normality is to construct a **Normal probability plot**.

## 2.2 TECHNOLOGY CORNERS

TI-Nspire instructions in Appendix B; HP Prime instructions on the book's Web site.

5. From  $z$ -scores to areas, and vice versa
6. Normal probability plots

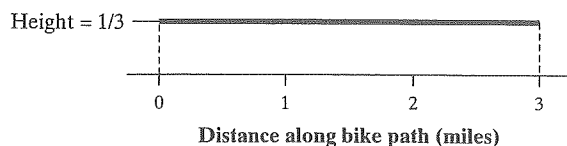
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## Section 2.2 Exercises

33. **Density curves** Sketch a density curve that might describe a distribution that is symmetric but has two peaks.
34. **Density curves** Sketch a density curve that might describe a distribution that has a single peak and is skewed to the left.

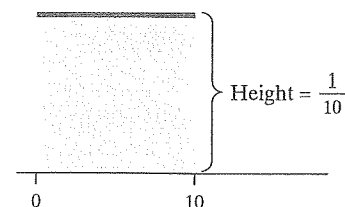
*Exercises 35 to 38 involve a special type of density curve—one that takes constant height (looks like a horizontal line) over some interval of values. This density curve describes a variable whose values are distributed evenly (uniformly) over some interval of values. We say that such a variable has a **uniform distribution**.*

35. **Biking accidents** Accidents on a level, 3-mile bike path occur uniformly along the length of the path. The figure below displays the density curve that describes the uniform distribution of accidents.



- (a) Explain why this curve satisfies the two requirements for a density curve.

- (b) The proportion of accidents that occur in the first mile of the path is the area under the density curve between 0 miles and 1 mile. What is this area?
- (c) Sue's property adjoins the bike path between the 0.8 mile mark and the 1.1 mile mark. What proportion of accidents happen in front of Sue's property? Explain.
36. **Where's the bus?** Sally takes the same bus to work every morning. The amount of time (in minutes) that she has to wait for the bus to arrive is described by the uniform distribution below.



- (a) Explain why this curve satisfies the two requirements for a density curve.
- (b) On what percent of days does Sally have to wait more than 8 minutes for the bus?

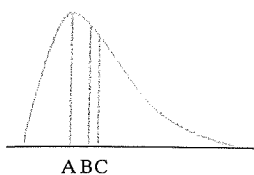


- (c) On what percent of days does Sally wait between 2.5 and 5.3 minutes for the bus?

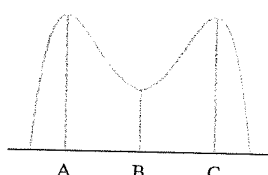
37. **Biking accidents** What is the mean  $\mu$  of the density curve pictured in Exercise 35? (That is, where would the curve balance?) What is the median? (That is, where is the point with area 0.5 on either side?)

38. **Where's the bus?** What is the mean  $\mu$  of the density curve pictured in Exercise 36? What is the median?

39. **Mean and median** The figure below displays two density curves, each with three points marked. At which of these points on each curve do the mean and the median fall?

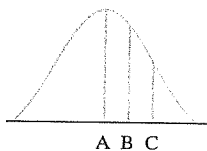


(a)

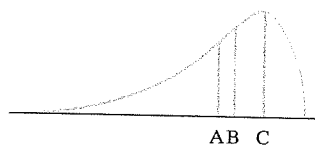


(b)

40. **Mean and median** The figure below displays two density curves, each with three points marked. At which of these points on each curve do the mean and the median fall?



(a)



(b)

41. **Men's heights** The distribution of heights of adult American men is approximately Normal with mean 69 inches and standard deviation 2.5 inches. Draw an accurate sketch of the distribution of men's heights. Be sure to label the mean, as well as the points 1, 2, and 3 standard deviations away from the mean on the horizontal axis.

42. **Potato chips** The distribution of weights of 9-ounce bags of a particular brand of potato chips is approximately Normal with mean  $\mu = 9.12$  ounces and standard deviation  $\sigma = 0.05$  ounce. Draw an accurate sketch of the distribution of potato chip bag weights. Be sure to label the mean, as well as the points 1, 2, and 3 standard deviations away from the mean on the horizontal axis.

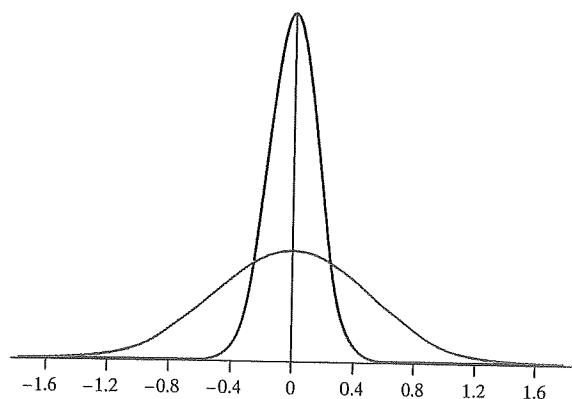
43. **Men's heights** Refer to Exercise 41. Use the 68–95–99.7 rule to answer the following questions. Show your work!

- Between what heights do the middle 95% of men fall?
- What percent of men are taller than 74 inches?
- What percent of men are between 64 and 66.5 inches tall?
- A height of 71.5 inches corresponds to what percentile of adult male American heights?

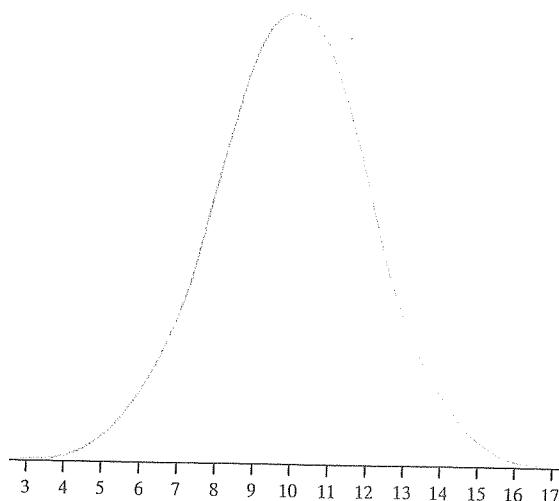
44. **Potato chips** Refer to Exercise 42. Use the 68–95–99.7 rule to answer the following questions. Show your work!

- Between what weights do the middle 68% of bags fall?
- What percent of bags weigh less than 9.02 ounces?
- What percent of 9-ounce bags of this brand of potato chips weigh between 8.97 and 9.17 ounces?
- A bag that weighs 9.07 ounces is at what percentile in this distribution?

45. **Estimating SD** The figure below shows two Normal curves, both with mean 0. Approximately what is the standard deviation of each of these curves?



46. **A Normal curve** Estimate the mean and standard deviation of the Normal density curve in the figure below.



For Exercises 47 to 50, use Table A to find the proportion of observations from the standard Normal distribution that satisfies each of the following statements. In each case, sketch a standard Normal curve and shade the area under the curve that is the answer to the question.


47. **Table A practice**

- $z < 2.85$
- $z > 2.85$
- $z > -1.66$
- $-1.66 < z < 2.85$

## 48. Table A practice

- (a)  $z < -2.46$       (c)  $0.89 < z < 2.46$   
 (b)  $z > 2.46$       (d)  $-2.95 < z < -1.27$

## pg 115 49. More Table A practice

-  (a)  $z$  is between  $-1.33$  and  $1.65$   
 (b)  $z$  is between  $0.50$  and  $1.79$

## 50. More Table A practice

- (a)  $z$  is between  $-2.05$  and  $0.78$   
 (b)  $z$  is between  $-1.11$  and  $-0.32$


For Exercises 51 and 52, use Table A to find the value  $z$  from the standard Normal distribution that satisfies each of the following conditions. In each case, sketch a standard Normal curve with your value of  $z$  marked on the axis.

## 51. Working backward

- (a) The 10th percentile.  
 (b) 34% of all observations are greater than  $z$ .

## 52. Working backward

- (a) The 63rd percentile.  
 (b) 75% of all observations are greater than  $z$ .

-  53. Length of pregnancies The length of human pregnancies from conception to birth varies according to a distribution that is approximately Normal with mean 266 days and standard deviation 16 days.

- pg 118 (a) At what percentile is a pregnancy that lasts 240 days (that's about 8 months)?

- pg 119 (b) What percent of pregnancies last between 240 and 270 days (roughly between 8 months and 9 months)?

- pg 120 (c) How long do the longest 20% of pregnancies last?

54. IQ test scores Scores on the Wechsler Adult Intelligence Scale (a standard IQ test) for the 20 to 34 age group are approximately Normally distributed with  $\mu = 110$  and  $\sigma = 25$ .

- (a) At what percentile is an IQ score of 150?  
 (b) What percent of people aged 20 to 34 have IQs between 125 and 150?  
 (c) MENSA is an elite organization that admits as members people who score in the top 2% on IQ tests. What score on the Wechsler Adult Intelligence Scale would an individual aged 20 to 34 have to earn to qualify for MENSA membership?

55. Put a lid on it! At some fast-food restaurants, customers who want a lid for their drinks get them from a large stack left near straws, napkins, and condiments.

The lids are made with a small amount of flexibility so they can be stretched across the mouth of the cup and then snugly secured. When lids are too small or too large, customers can get very frustrated, especially if they end up spilling their drinks. At one particular restaurant, large drink cups require lids with a "diameter" of between 3.95 and 4.05 inches. The restaurant's lid supplier claims that the diameter of their large lids follows a Normal distribution with mean 3.98 inches and standard deviation 0.02 inches. Assume that the supplier's claim is true.

- (a) What percent of large lids are too small to fit? Show your method.  
 (b) What percent of large lids are too big to fit? Show your method.  
 (c) Compare your answers to parts (a) and (b). Does it make sense for the lid manufacturer to try to make one of these values larger than the other? Why or why not?
56. I think I can! An important measure of the performance of a locomotive is its "adhesion," which is the locomotive's pulling force as a multiple of its weight. The adhesion of one 4400-horsepower diesel locomotive varies in actual use according to a Normal distribution with mean  $\mu = 0.37$  and standard deviation  $\sigma = 0.04$ .
- (a) For a certain small train's daily route, the locomotive needs to have an adhesion of at least 0.30 for the train to arrive at its destination on time. On what proportion of days will this happen? Show your method.  
 (b) An adhesion greater than 0.50 for the locomotive will result in a problem because the train will arrive too early at a switch point along the route. On what proportion of days will this happen? Show your method.  
 (c) Compare your answers to parts (a) and (b). Does it make sense to try to make one of these values larger than the other? Why or why not?
57. Put a lid on it! Refer to Exercise 55. The supplier is considering two changes to reduce the percent of its large-cup lids that are too small to fit. One strategy is to adjust the mean diameter of its lids. Another option is to alter the production process, thereby decreasing the standard deviation of the lid diameters.
- (a) If the standard deviation remains at  $\sigma = 0.02$  inches, at what value should the supplier set the mean diameter of its large-cup lids so that only 1% are too small to fit? Show your method.  
 (b) If the mean diameter stays at  $\mu = 3.98$  inches, what value of the standard deviation will result in only 1% of lids that are too small to fit? Show your method.



- (c) Which of the two options in parts (a) and (b) do you think is preferable? Justify your answer. (Be sure to consider the effect of these changes on the percent of lids that are too large to fit.)

58. **I think I can!** Refer to Exercise 56. The locomotive's manufacturer is considering two changes that could reduce the percent of times that the train arrives late. One option is to increase the mean adhesion of the locomotive. The other possibility is to decrease the variability in adhesion from trip to trip, that is, to reduce the standard deviation.

- (a) If the standard deviation remains at  $\sigma = 0.04$ , to what value must the manufacturer change the mean adhesion of the locomotive to reduce its proportion of late arrivals to only 2% of days? Show your work.
- (b) If the mean adhesion stays at  $\mu = 0.37$ , how much must the standard deviation be decreased to ensure that the train will arrive late only 2% of the time? Show your work.
- (c) Which of the two options in parts (a) and (b) do you think is preferable? Justify your answer. (Be sure to consider the effect of these changes on the percent of days that the train arrives early to the switch point.)

59. **Deciles** The deciles of any distribution are the values at the 10th, 20th, . . . , 90th percentiles. The first and last deciles are the 10th and the 90th percentiles, respectively.

- (a) What are the first and last deciles of the standard Normal distribution?
- (b) The heights of young women are approximately Normal with mean 64.5 inches and standard deviation 2.5 inches. What are the first and last deciles of this distribution? Show your work.

60. **Outliers** The percent of the observations that are classified as outliers by the  $1.5 \times IQR$  rule is the same in any Normal distribution. What is this percent? Show your method clearly.

61. **Flight times** An airline flies the same route at the same time each day. The flight time varies according to a Normal distribution with unknown mean and standard deviation. On 15% of days, the flight takes more than an hour. On 3% of days, the flight lasts 75 minutes or more. Use this information to determine the mean and standard deviation of the flight time distribution.

62. **Brush your teeth** The amount of time Ricardo spends brushing his teeth follows a Normal distribution with unknown mean and standard deviation. Ricardo spends less than one minute brushing his teeth about 40% of the time. He spends more than

two minutes brushing his teeth 2% of the time. Use this information to determine the mean and standard deviation of this distribution.

63. **Sharks** Here are the lengths in feet of 44 great white sharks:<sup>11</sup>



18.7	12.3	18.6	16.4	15.7	18.3	14.6	15.8	14.9	17.6	12.1
16.4	16.7	17.8	16.2	12.6	17.8	13.8	12.2	15.2	14.7	12.4
13.2	15.8	14.3	16.6	9.4	18.2	13.2	13.6	15.3	16.1	13.5
19.1	16.2	22.8	16.8	13.6	13.2	15.7	19.7	18.7	13.2	16.8

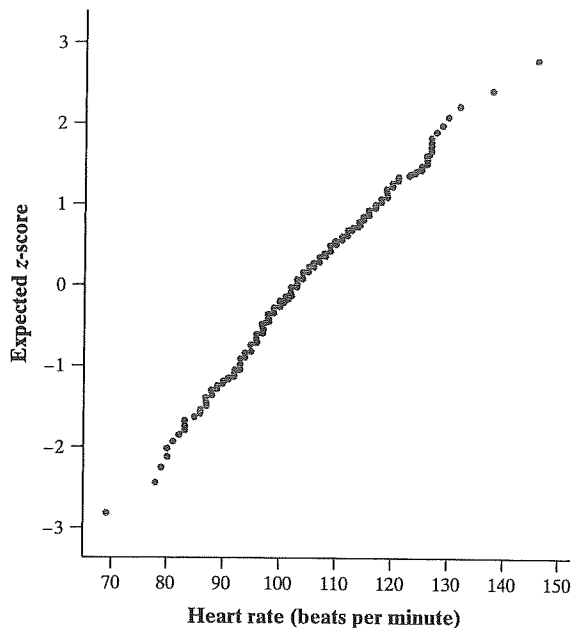
- (a) Enter these data into your calculator and make a histogram. Include a sketch of the graph on your paper. Then calculate one-variable statistics. Describe the shape, center, and spread of the distribution of shark lengths.
- (b) Calculate the percent of observations that fall within 1, 2, and 3 standard deviations of the mean. How do these results compare with the 68–95–99.7 rule?
- (c) Use your calculator to construct a Normal probability plot. Include a sketch of the graph on your paper. Interpret this plot.
- (d) Having inspected the data from several different perspectives, do you think these data are approximately Normal? Write a brief summary of your assessment that combines your findings from parts (a) through (c).

64. **Density of the earth** In 1798, the English scientist Henry Cavendish measured the density of the earth several times by careful work with a torsion balance. The variable recorded was the density of the earth as a multiple of the density of water. Here are Cavendish's 29 measurements:<sup>12</sup>

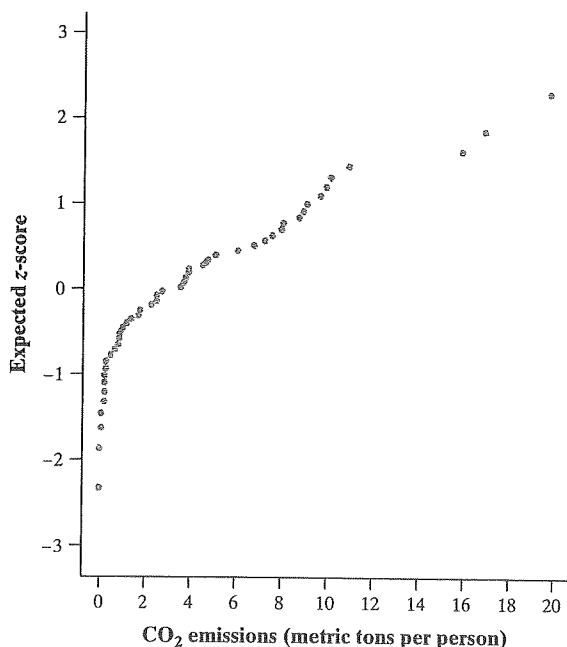
5.50	5.61	4.88	5.07	5.26	5.55	5.36	5.29	5.58	5.65
5.57	5.53	5.62	5.29	5.44	5.34	5.79	5.10	5.27	5.39
5.42	5.47	5.63	5.34	5.46	5.30	5.75	5.68	5.85	

- (a) Enter these data into your calculator and make a histogram. Include a sketch of the graph on your paper. Then calculate one-variable statistics. Describe the shape, center, and spread of the distribution of density measurements.
- (b) Calculate the percent of observations that fall within 1, 2, and 3 standard deviations of the mean. How do these results compare with the 68–95–99.7 rule?
- (c) Use your calculator to construct a Normal probability plot. Include a sketch of the graph on your paper. Interpret this plot.
- (d) Having inspected the data from several different perspectives, do you think these data are approximately Normal? Write a brief summary of your assessment that combines your findings from parts (a) through (c).

65. **Runners' heart rates** The figure below is a Normal probability plot of the heart rates of 200 male runners after six minutes of exercise on a treadmill.<sup>13</sup> The distribution is close to Normal. How can you see this? Describe the nature of the small deviations from Normality that are visible in the plot.



66. **Carbon dioxide emissions** The figure below is a Normal probability plot of the emissions of carbon dioxide per person in 48 countries.<sup>14</sup> In what ways is this distribution non-Normal?



67. **Is Michigan Normal?** We collected data on the tuition charged by colleges and universities in Michigan. Here are some numerical summaries for the data:

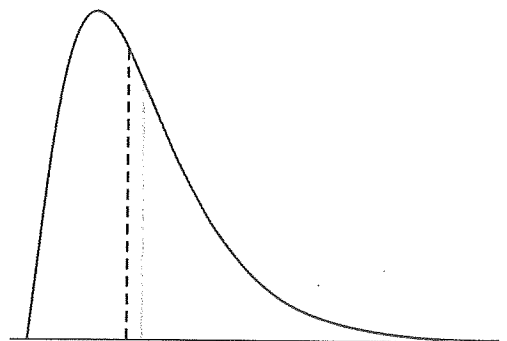
Mean	Std. Dev.	Min	Max
10614	8049	1873	30823

Based on the relationship between the mean, standard deviation, minimum, and maximum, is it reasonable to believe that the distribution of Michigan tuitions is approximately Normal? Explain.

68. **Weights aren't Normal** The heights of people of the same gender and similar ages follow Normal distributions reasonably closely. Weights, on the other hand, are not Normally distributed. The weights of women aged 20 to 29 have mean 141.7 pounds and median 133.2 pounds. The first and third quartiles are 118.3 pounds and 157.3 pounds. What can you say about the shape of the weight distribution? Why?

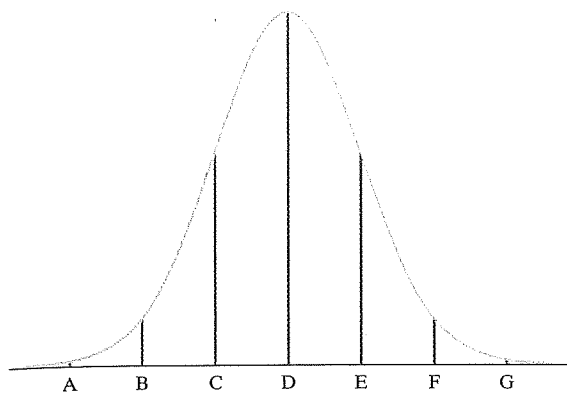
*Multiple choice: Select the best answer for Exercises 69 to 74.*

69. Two measures of center are marked on the density curve shown. Which of the following is correct?

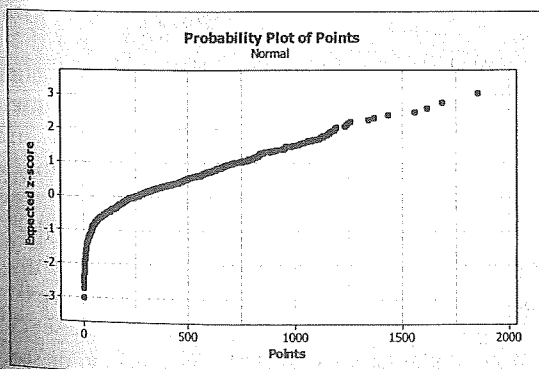


- (a) The median is at the yellow line and the mean is at the red line.
- (b) The median is at the red line and the mean is at the yellow line.
- (c) The mode is at the red line and the median is at the yellow line.
- (d) The mode is at the yellow line and the median is at the red line.
- (e) The mode is at the red line and the mean is at the yellow line.

*Exercises 70 to 72 refer to the following setting.* The weights of laboratory cockroaches follow a Normal distribution with mean 80 grams and standard deviation 2 grams. The following figure is the Normal curve for this distribution of weights.

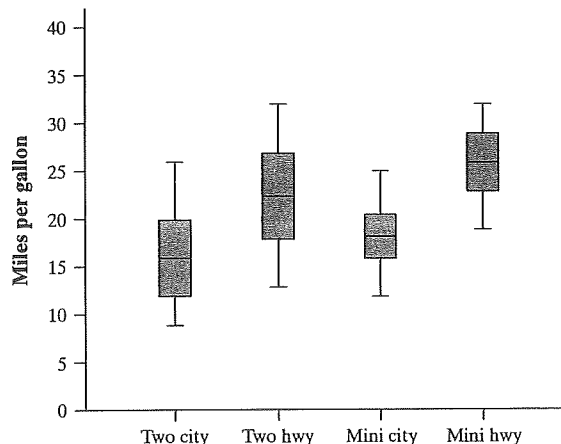


70. Point C on this Normal curve corresponds to  
 (a) 84 grams. (c) 78 grams. (e) 74 grams.  
 (b) 82 grams. (d) 76 grams.
71. About what percent of the cockroaches have weights between 76 and 84 grams?  
 (a) 99.7% (c) 68% (e) 34%  
 (b) 95% (d) 47.5%
72. About what proportion of the cockroaches will have weights greater than 83 grams?  
 (a) 0.0228 (c) 0.1587 (e) 0.0772  
 (b) 0.0668 (d) 0.9332
73. A different species of cockroach has weights that follow a Normal distribution with a mean of 50 grams. After measuring the weights of many of these cockroaches, a lab assistant reports that 14% of the cockroaches weigh more than 55 grams. Based on this report, what is the approximate standard deviation of weights for this species of cockroaches?  
 (a) 4.6 (d) 14.0  
 (b) 5.0 (e) Cannot determine without more information.  
 (c) 6.2
74. The following Normal probability plot shows the distribution of points scored for the 551 players in the 2011–2012 NBA season.



If the distribution of points was displayed in a histogram, what would be the best description of the histogram's shape?

- (a) Approximately Normal  
 (b) Symmetric but not approximately Normal  
 (c) Skewed left  
 (d) Skewed right  
 (e) Cannot be determined
75. Gas it up! (1.3) Interested in a sporty car? Worried that it might use too much gas? The Environmental Protection Agency lists most such vehicles in its "two-seater" or "minicompact" categories. The figure shows boxplots for both city and highway gas mileages for our two groups of cars. Write a few sentences comparing these distributions.



76. Python eggs (1.1) How is the hatching of water python eggs influenced by the temperature of the snake's nest? Researchers assigned newly laid eggs to one of three temperatures: hot, neutral, or cold. Hot duplicates the extra warmth provided by the mother python, and cold duplicates the absence of the mother. Here are the data on the number of eggs and the number that hatched:<sup>15</sup>

	Cold	Neutral	Hot
Number of eggs	27	56	104
Number hatched	16	38	75

- (a) Make a two-way table of temperature by outcome (hatched or not).  
 (b) Calculate the percent of eggs in each group that hatched. The researchers believed that eggs would be less likely to hatch in cold water. Do the data support that belief?