

Section 11.1 Exercises

1. **Aw, nuts!** A company claims that each batch of its deluxe mixed nuts contains 52% cashews, 27% almonds, 13% macadamia nuts, and 8% brazil nuts. To test this claim, a quality-control inspector takes a random sample of 150 nuts from the latest batch. The one-way table below displays the sample data.

Nut:	Cashew	Almond	Macadamia	Brazil
Count:	83	29	20	18

- (a) State appropriate hypotheses for performing a test of the company's claim.
- (b) Calculate the expected counts for each type of nut. Show your work.
2. **Roulette** Casinos are required to verify that their games operate as advertised. American roulette wheels have 38 slots—18 red, 18 black, and 2 green. In one casino, managers record data from a random sample of 200 spins of one of their American roulette wheels. The one-way table below displays the results.

Color:	Red	Black	Green
Count:	85	99	16

- (a) State appropriate hypotheses for testing whether these data give convincing evidence that the distribution of outcomes on this wheel is not what it should be.
- (b) Calculate the expected counts for each color. Show your work.
3. **Aw, nuts!** Calculate the chi-square statistic for the data in Exercise 1. Show your work.
4. **Roulette** Calculate the chi-square statistic for the data in Exercise 2. Show your work.
5. **Aw, nuts!** Refer to Exercises 1 and 3.
- (a) Confirm that the expected counts are large enough to use a chi-square distribution to calculate the P -value. What degrees of freedom should you use?
- (b) Sketch a graph like Figure 11.4 (page 685) that shows the P -value.
- (c) Use Table C to find the P -value. Then use your calculator's $\chi^2\text{cdf}$ command.
- (d) What conclusion would you draw about the company's claimed distribution for its deluxe mixed nuts? Justify your answer.

6. **Roulette** Refer to Exercises 2 and 4.

- (a) Confirm that the expected counts are large enough to use a chi-square distribution to calculate the P -value. What degrees of freedom should you use?
- (b) Sketch a graph like Figure 11.4 (page 685) that shows the P -value.
- (c) Use Table C to find the P -value. Then use your calculator's $\chi^2\text{cdf}$ command.
- (d) What conclusion would you draw about whether or not the roulette wheel is operating correctly? Justify your answer.

7. **Birds in the trees** Researchers studied the behavior of birds that were searching for seeds and insects in an Oregon forest. In this forest, 54% of the trees were Douglas firs, 40% were ponderosa pines, and 6% were other types of trees. At a randomly selected time during the day, the researchers observed 156 red-breasted nuthatches: 70 were seen in Douglas firs, 79 in ponderosa pines, and 7 in other types of trees.² Do these data provide convincing evidence that nuthatches prefer particular types of trees when they're searching for seeds and insects?

8. **Seagulls by the seashore** Do seagulls show a preference for where they land? To answer this question, biologists conducted a study in an enclosed outdoor space with a piece of shore whose area was made up of 56% sand, 29% mud, and 15% rocks. The biologists chose 200 seagulls at random. Each seagull was released into the outdoor space on its own and observed until it landed somewhere on the piece of shore. In all, 128 seagulls landed on the sand, 61 landed in the mud, and 11 landed on the rocks. Do these data provide convincing evidence that seagulls show a preference for where they land?

9. **No chi-square** A school's principal wants to know if students spend about the same amount of time on homework each night of the week. She asks a random sample of 50 students to keep track of their homework time for a week. The following table displays the average amount of time (in minutes) students reported per night:

Night:	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Average time:	130	108	115	104	99	37	62

Explain carefully why it would not be appropriate to perform a chi-square test for goodness of fit using these data.

10. **No chi-square** The principal in Exercise 9 also asked the random sample of students to record whether they did all of the homework that was assigned on each of the five school days that week. Here are the data:

School day:	Monday	Tuesday	Wednesday	Thursday	Friday
No. who did homework:	34	29	32	28	19

Explain carefully why it would not be appropriate to perform a chi-square test for goodness of fit using these data.

11. **Benford's law** Faked numbers in tax returns, invoices, or expense account claims often display patterns that aren't present in legitimate records. Some patterns are obvious and easily avoided by a clever crook. Others are more subtle. It is a striking fact that the first digits of numbers in legitimate records often follow a model known as Benford's law.³ Call the first digit of a randomly chosen record X for short. Benford's law gives this probability model for X (note that a first digit can't be 0):

First digit:	1	2	3	4	5	6	7	8	9
Probability:	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

A forensic accountant who is familiar with Benford's law inspects a random sample of 250 invoices from a company that is accused of committing fraud. The table below displays the sample data.

First digit:	1	2	3	4	5	6	7	8	9
Count:	61	50	43	34	25	16	7	8	6

- (a) Are these data inconsistent with Benford's law? Carry out an appropriate test at the $\alpha = 0.05$ level to support your answer. If you find a significant result, perform a follow-up analysis.
- (b) Describe a Type I error and a Type II error in this setting, and give a possible consequence of each. Which do you think is more serious?
12. **Housing** According to the Census Bureau, the distribution by ethnic background of the New York City population in a recent year was

Hispanic: 28% Black: 24% White: 35%
Asian: 12% Others: 1%

The manager of a large housing complex in the city wonders whether the distribution by race of the

complex's residents is consistent with the population distribution. To find out, she records data from a random sample of 800 residents. The table below displays the sample data.⁴

Race:	Hispanic	Black	White	Asian	Other
Count:	212	202	270	94	22

Are these data significantly different from the city's distribution by race? Carry out an appropriate test at the $\alpha = 0.05$ level to support your answer. If you find a significant result, perform a follow-up analysis.

13. **Skittles** Statistics teacher Jason Molesky contacted Mars, Inc., to ask about the color distribution for Skittles candies. Here is an excerpt from the response he received: "The original flavor blend for the SKITTLES BITE SIZE CANDIES is lemon, lime, orange, strawberry and grape. They were chosen as a result of consumer preference tests we conducted. The flavor blend is 20 percent of each flavor."
- (a) State appropriate hypotheses for a significance test of the company's claim.
- (b) Find the expected counts for a bag of Skittles with 60 candies.
- (c) How large a χ^2 statistic would you need to have significant evidence against the company's claim at the $\alpha = 0.05$ level? At the $\alpha = 0.01$ level?
- (d) Create a set of observed counts for a bag with 60 candies that gives a P -value between 0.01 and 0.05. Show the calculation of your chi-square statistic.
14. **Is your random number generator working?** Use your calculator's RandInt function to generate 200 digits from 0 to 9 and store them in a list.
- (a) State appropriate hypotheses for a chi-square test for goodness of fit to determine whether your calculator's random number generator gives each digit an equal chance to be generated.
- (b) Carry out a test at the $\alpha = 0.05$ significance level.
- For parts (c) and (d), assume that the students' random number generators are all working properly.
- (c) What is the probability that a student who does this exercise will make a Type I error?
- (d) Suppose that 25 students in an AP Statistics class independently do this exercise for homework. Find the probability that at least one of them makes a Type I error.

15. **What's your sign?** The University of Chicago's General Social Survey (GSS) is the nation's most important social science sample survey. For reasons known





only to social scientists, the GSS regularly asks a random sample of people their astrological sign. Here are the counts of responses from a recent GSS:

Sign:	Aries	Taurus	Gemini	Cancer	Leo	Virgo
Count:	321	360	367	374	383	402
Sign:	Libra	Scorpio	Sagittarius	Capricorn	Aquarius	Pisces
Count:	392	329	331	354	376	355

If births are spread uniformly across the year, we expect all 12 signs to be equally likely. Do these data provide convincing evidence that all 12 signs are not equally likely? If you find a significant result, perform a follow-up analysis.

6. **Munching Froot Loops** Kellogg's Froot Loops cereal comes in six fruit flavors: orange, lemon, cherry, raspberry, blueberry, and lime. Charise poured out her morning bowl of cereal and methodically counted the number of cereal pieces of each flavor. Here are her data:

Flavor:	Orange	Lemon	Cherry	Raspberry	Blueberry	Lime
Count:	28	21	16	25	14	16

Do these data provide convincing evidence that Kellogg's Froot Loops do not contain an equal proportion of each flavor? If you find a significant result, perform a follow-up analysis.

7. **Mendel and the peas** Gregor Mendel (1822–1884), an Austrian monk, is considered the father of genetics. Mendel studied the inheritance of various traits in pea plants. One such trait is whether the pea is smooth or wrinkled. Mendel predicted a ratio of 3 smooth peas for every 1 wrinkled pea. In one experiment, he observed 423 smooth and 133 wrinkled peas. Assume that the conditions for inference were met. Carry out an appropriate test of the genetic model that Mendel predicted. What do you conclude?
8. **You say tomato** The paper "Linkage Studies of the Tomato" (*Transactions of the Canadian Institute*, 1931) reported the following data on phenotypes resulting from crossing tall cut-leaf tomatoes with dwarf potato-leaf tomatoes. We wish to investigate whether the following frequencies are consistent with genetic laws, which state that the phenotypes should occur in the ratio 9:3:3:1.

Phenotype:	Tall cut	Tall potato	Dwarf cut	Dwarf potato
Frequency:	926	288	293	104

Assume that the conditions for inference were met. Carry out an appropriate test of the proposed genetic model. What do you conclude?

Multiple choice: Select the best answer for Exercises 19 to 22.

Exercises 19 to 21 refer to the following setting. The manager of a high school cafeteria is planning to offer several new types of food for student lunches in the following school year. She wants to know if each type of food will be equally popular so she can start ordering supplies and making other plans. To find out, she selects a random sample of 100 students and asks them, "Which type of food do you prefer: Asian food, Mexican food, pizza, or hamburgers?" Here are her data:

Type of Food:	Asian	Mexican	Pizza	Hamburgers
Count:	18	22	39	21

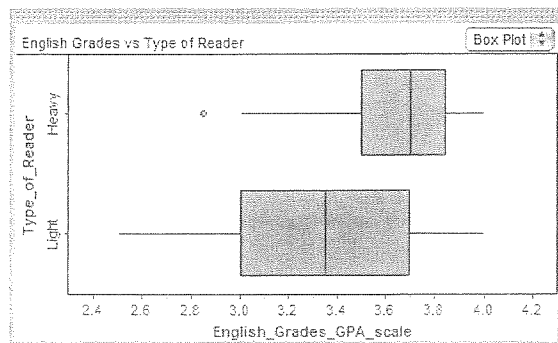
19. An appropriate null hypothesis to test whether the food choices are equally popular is
- $H_0: \mu = 25$, where μ = the mean number of students that prefer each type of food.
 - $H_0: p = 0.25$, where p = the proportion of all students who prefer Asian food.
 - $H_0: n_A = n_M = n_P = n_H = 25$, where n_A is the number of students in the school who would choose Asian food, and so on.
 - $H_0: p_A = p_M = p_P = p_H = 0.25$, where p_A is the proportion of students in the school who would choose Asian food, and so on.
 - $H_0: \hat{p}_A = \hat{p}_M = \hat{p}_P = \hat{p}_H = 0.25$, where \hat{p}_A is the proportion of students in the sample who chose Asian food, and so on.
20. The chi-square statistic is
- $\frac{(18 - 25)^2}{25} + \frac{(22 - 25)^2}{25} + \frac{(39 - 25)^2}{25} + \frac{(21 - 25)^2}{25}$
 - $\frac{(25 - 18)^2}{18} + \frac{(25 - 22)^2}{22} + \frac{(25 - 39)^2}{39} + \frac{(25 - 21)^2}{21}$
 - $\frac{(18 - 25)}{25} + \frac{(22 - 25)}{25} + \frac{(39 - 25)}{25} + \frac{(21 - 25)}{25}$
 - $\frac{(18 - 25)^2}{100} + \frac{(22 - 25)^2}{100} + \frac{(39 - 25)^2}{100} + \frac{(21 - 25)^2}{100}$
 - $\frac{(0.18 - 0.25)^2}{0.25} + \frac{(0.22 - 0.25)^2}{0.25} + \frac{(0.39 - 0.25)^2}{0.25} + \frac{(0.21 - 0.25)^2}{0.25}$
21. The P -value for a chi-square test for goodness of fit is 0.0129. Which of the following is the most appropriate conclusion?

- (a) Because 0.0129 is less than $\alpha = 0.05$, reject H_0 . There is convincing evidence that the food choices are equally popular.
- (b) Because 0.0129 is less than $\alpha = 0.05$, reject H_0 . There is not convincing evidence that the food choices are equally popular.
- (c) Because 0.0129 is less than $\alpha = 0.05$, reject H_0 . There is convincing evidence that the food choices are not equally popular.
- (d) Because 0.0129 is less than $\alpha = 0.05$, fail to reject H_0 . There is not convincing evidence that the food choices are equally popular.
- (e) Because 0.0129 is less than $\alpha = 0.05$, fail to reject H_0 . There is convincing evidence that the food choices are equally popular.

22. Which of the following is *false*?

- (a) A chi-square distribution with k degrees of freedom is more right-skewed than a chi-square distribution with $k + 1$ degrees of freedom.
- (b) A chi-square distribution never takes negative values.
- (c) The degrees of freedom for a chi-square test is determined by the sample size.
- (d) $P(\chi^2 > 10)$ is greater when $df = k + 1$ than when $df = k$.
- (e) The area under a chi-square density curve is always equal to 1.

Exercises 23 through 25 refer to the following setting. Do students who read more books for pleasure tend to earn higher grades in English? The boxplots below show data from a simple random sample of 79 students at a large high school. Students were classified as light readers if they read fewer than 3 books for pleasure per year. Otherwise, they were classified as heavy readers. Each student's average English grade for the previous two marking periods was converted to a GPA scale where $A+ = 4.3$, $A = 4.0$, $A- = 3.7$, $B+ = 3.3$, and so on.

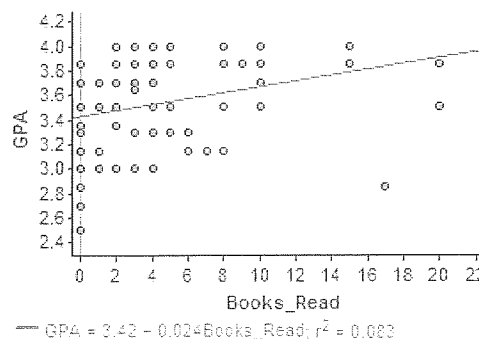


23. Reading and grades (1.5) Write a few sentences comparing the distributions of English grades for light and heavy readers.

24. Reading and grades (10.2) Summary statistics for the two groups from Minitab are provided below.

Type of reader	N	Mean	StDev	SE Mean
Heavy	47	3.640	0.324	0.047
Light	32	3.356	0.380	0.067

- (a) Explain why it is acceptable to use two-sample t procedures in this setting.
- (b) Construct and interpret a 95% confidence interval for the difference in the mean English grade for light and heavy readers.
- (c) Does the interval in part (b) provide convincing evidence that reading more causes a difference in students' English grades? Justify your answer.
25. Reading and grades (3.2) The Fathom scatterplot below shows the number of books read and the English grade for all 79 students in the study. A least-squares regression line has been added to the graph.



- (a) Interpret the meaning of the slope and y intercept in context.
- (b) The student who reported reading 17 books for pleasure had an English GPA of 2.85. Find this student's residual and interpret this value in context.
- (c) How strong is the relationship between English grades and number of books read? Give appropriate evidence to support your answer.
26. Yahtzee (5.3, 6.3) In the game of Yahtzee, 5 six-sided dice are rolled simultaneously. To get a Yahtzee, the player must get the same number on all 5 dice.
- (a) Luis says that the probability of getting a Yahtzee in one roll of the dice is $\left(\frac{1}{6}\right)^5$. Explain why Luis is wrong.
- (b) Nassir decides to keep rolling all 5 dice until he gets a Yahtzee. He is surprised when he still hasn't gotten a Yahtzee after 25 rolls. Should he be? Calculate an appropriate probability to support your answer.