When conditions are met, the **two-sample z test for $p_1 - p_2$** uses the test statistic

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\frac{\hat{p}_C(1-\hat{p}_C)}{n_1} + \frac{\hat{p}_C(1-\hat{p}_C)}{n_2}}}$$

with P-values calculated from the standard Normal distribution.

- Inference about the difference $p_1 - p_2$ in the effectiveness of two treatments in a completely randomized experiment is based on the **randomization distribution** of $\hat{p}_1 - \hat{p}_2$. When conditions are met, our usual inference procedures based on the sampling distribution of $\hat{p}_1 - \hat{p}_2$ will be approximately correct.
- Be sure to follow the four-step process whenever you construct a confidence interval or perform a significance test for comparing two proportions.

STEP 4

10.1 TECHNOLOGY CORNERS

TI-Nspire Instructions in Appendix B; HP Prime instructions on the book's Web site.

Section 10.1 Exercises

STEP 4

Remember: We are no longer reminding you to use the four-step process in exercises that require you to perform inference.

1. **Goldfish** Refer to the example on page 615. Suppose that your teacher decides to take SRSs of 100 crackers from both bags instead.

(a) What is the shape of the sampling distribution of $\hat{p}_1 - \hat{p}_2$? Why?

(b) Find the mean of the sampling distribution. Show your work.

(c) Find the standard deviation of the sampling distribution. Show your work.

2. **Homework** Refer to page 612. Suppose that both school counselors decide to take SRSs of 150 students instead.

(a) What is the shape of the sampling distribution of $\hat{p}_1 - \hat{p}_2$? Why?

(b) Find the mean of the sampling distribution. Show your work.

(c) Find the standard deviation of the sampling distribution. Show your work.

3. **I want red!** A candy maker offers Child and Adult bags of jelly beans with different color mixes. The company claims that the Child mix has 30% red jelly beans, while the Adult mix contains 15% red jelly beans. Assume that the candy maker's claim is true. Suppose we take a random sample of 50 jelly beans from the Child mix and a separate random sample of 100 jelly beans from the Adult mix. Let $\hat{p}_C$ and $\hat{p}_A$ be the sample proportions of red jelly beans from the Child and Adult mixes, respectively.

(a) What is the shape of the sampling distribution of $\hat{p}_C - \hat{p}_A$? Why?

(b) Find the mean of the sampling distribution. Show your work.

(c) Find the standard deviation of the sampling distribution. Show your work.

4. **Literacy** A researcher reports that 80% of high school graduates, but only 40% of high school dropouts, would pass a basic literacy test.[5] Assume that the researcher's claim is true. Suppose we give

a basic literacy test to a random sample of 60 high school graduates and a separate random sample of 75 high school dropouts. Let $\hat{p}_G$ and $\hat{p}_D$ be the sample proportions of graduates and dropouts, respectively, who pass the test.

(a) What is the shape of the sampling distribution of $\hat{p}_G - \hat{p}_D$? Why?

(b) Find the mean of the sampling distribution. Show your work.

(c) Find the standard deviation of the sampling distribution. Show your work.

Explain why the conditions for constructing a two-sample z interval for $p_1 - p_2$ are not met in the settings of Exercises 5 through 8.

5. **Don't drink the water!** The movie *A Civil Action* (Touchstone Pictures, 1998) tells the story of a major legal battle that took place in the small town of Woburn, Massachusetts. A town well that supplied water to eastern Woburn residents was contaminated by industrial chemicals. During the period that residents drank water from this well, 16 of the 414 babies born had birth defects. On the west side of Woburn, 3 of the 228 babies born during the same time period had birth defects.

6. **In-line skaters** A study of injuries to in-line skaters used data from the National Electronic Injury Surveillance System, which collects data from a random sample of hospital emergency rooms. The researchers interviewed 161 people who came to emergency rooms with injuries from in-line skating. Wrist injuries (mostly fractures) were the most common.[6] The interviews found that 53 people were wearing wrist guards and 6 of these had wrist injuries. Of the 108 who did not wear wrist guards, 45 had wrist injuries.

7. **Shrubs and fire** Fire is a serious threat to shrubs in dry climates. Some shrubs can resprout from their roots after their tops are destroyed. One study of resprouting took place in a dry area of Mexico.[7] The investigators randomly assigned shrubs to treatment and control groups. They clipped the tops of all the shrubs. They then applied a propane torch to the stumps of the treatment group to simulate a fire. All 12 of the shrubs in the treatment group resprouted. Only 8 of the 12 shrubs in the control group resprouted.

8. **Broken crackers** We don't like to find broken crackers when we open the package. How can makers reduce breaking? One idea is to microwave the crackers for 30 seconds right after baking them. Breaks start as hairline cracks called "checking." Randomly assign 65 newly baked crackers to the microwave and another 65 to a control group that is not microwaved. After one day, none of the microwave group and 16 of the control group show checking.[8]

9. **Who tweets?** Do younger people use Twitter more often than older people? In a random sample of 316 adult Internet users aged 18 to 29, 26% used Twitter. In a separate random sample of 532 adult Internet users aged 30 to 49, 14% used Twitter.[9]

(a) Calculate the standard error of the sampling distribution of the difference in the sample proportions (younger adults − older adults). What information does this value provide?

(b) Construct and interpret a 90% confidence interval for the difference between the true proportions of adult Internet users in these age groups who use Twitter.

10. **Listening to rap** Is rap music more popular among young blacks than among young whites? A sample survey compared 634 randomly chosen blacks aged 15 to 25 with 567 randomly selected whites in the same age group. It found that 368 of the blacks and 130 of the whites listened to rap music every day.[10]

(a) Calculate the standard error of the sampling distribution of the difference in the sample proportions (blacks − whites). What information does this value provide?

(b) Construct and interpret a 95% confidence interval for the difference between the proportions of black and white young people who listen to rap every day.

11. **Young adults living at home** A surprising number of young adults (ages 19 to 25) still live in their parents' homes. A random sample by the National Institutes of Health included 2253 men and 2629 women in this age group.[11] The survey found that 986 of the men and 923 of the women lived with their parents.

(a) Construct and interpret a 99% confidence interval for the difference in the true proportions of men and women aged 19 to 25 who live in their parents' homes.

(b) Does your interval from part (a) give convincing evidence of a difference between the population proportions? Explain.

12. **Fear of crime** The elderly fear crime more than younger people, even though they are less likely to be victims of crime. One study recruited separate random samples of 56 black women and 63 black men over the age of 65 from Atlantic City, New Jersey. Of the women, 27 said they "felt vulnerable" to crime; 46 of the men said this.[12]

(a) Construct and interpret a 90% confidence interval for the difference in the true proportions of black women and black men in Atlantic City who would say they felt vulnerable to crime.

(b) Does your interval from part (a) give convincing evidence of a difference between the population proportions? Explain.

13. **Who owns iPods?** As part of the Pew Internet and American Life Project, researchers surveyed a random sample of 800 teens and a separate random sample of 400 young adults. For the teens, 79% said that they own an iPod or MP3 player. For the young adults, this figure was 67%. Do the data give convincing evidence of a difference in the proportions of all teens and young adults who would say that they own an iPod or MP3 player? State appropriate hypotheses for a test to answer this question. Define any parameters you use.

14. **Steroids in high school** A study by the National Athletic Trainers Association surveyed random samples of 1679 high school freshmen and 1366 high school seniors in Illinois. Results showed that 34 of the freshmen and 24 of the seniors had used anabolic steroids. Steroids, which are dangerous, are sometimes used in an attempt to improve athletic performance.[13] Do the data give convincing evidence of a difference in the proportion of all Illinois high school freshmen and seniors who have used anabolic steroids? State appropriate hypotheses for a test to answer this question. Define any parameters you use.

15. **Who owns iPods?** Refer to Exercise 13. Carry out a significance test at the $\alpha = 0.05$ level.

16. **Steroids in high school** Refer to Exercise 14. Carry out a significance test at the $\alpha = 0.05$ level.

17. **Who owns iPods?** Refer to Exercise 13. Construct and interpret a 95% confidence interval for the difference between the population proportions. Explain how the confidence interval is consistent with the results of the test in Exercise 15.

18. **Steroids in high school** Refer to Exercise 14. Construct and interpret a 95% confidence interval for the difference between the population proportions. Explain how the confidence interval is consistent with the results of the test in Exercise 16.

19. **Children make choices** Many new products introduced into the market are targeted toward children. The choice behavior of children with regard to new products is of particular interest to companies that design marketing strategies for these products. As part of one study, randomly selected children in different age groups were compared on their ability to sort new products into the correct product category (milk or juice).[14] Here are some of the data:

Age group	N	Number who sorted correctly
4- to 5-year-olds	50	10
6- to 7-year-olds	53	28

Did a significantly higher proportion of the 6- to 7-year-olds than the 4- to 5-year-olds sort correctly? Give appropriate evidence to justify your answer.

20. **Marriage and status** "Would you marry a person from a lower social class than your own?" Researchers asked this question of a random sample of 385 black, never-married college students. Of the 149 men in the sample, 91 said "Yes." Among the 236 women, 117 said "Yes."[15] Did a significantly higher proportion of the men than the women who were surveyed say "Yes"? Give appropriate evidence to justify your answer.

21. **Driving school** A driving school owner believes that Instructor A is more effective than Instructor B at preparing students to pass the state's driver's license exam. An incoming class of 100 students is randomly assigned to two groups, each of size 50. One group is taught by Instructor A; the other is taught by Instructor B. At the end of the course, 30 of Instructor A's students and 22 of Instructor B's students pass the state exam.

(a) Do these results give convincing evidence at the $\alpha = 0.05$ level that Instructor A is more effective?

(b) Describe a Type I and a Type II error in this setting. Which error could you have made in part (a)?

22. **Preventing strokes** Aspirin prevents blood from clotting and so helps prevent strokes. The Second European Stroke Prevention Study asked whether adding another anticlotting drug, named dipyridamole, would be more effective for patients who had already had a stroke. Here are the data on strokes during the two years of the study:[16]

	Number of patients	Number of strokes
Aspirin alone	1649	206
Aspirin + dipyridamole	1650	157

The study was a randomized comparative experiment.

(a) Is there convincing evidence at the $\alpha = 0.05$ level that adding dipyridamole helps reduce the risk of stroke?

(b) Describe a Type I and a Type II error in this setting. Which is more serious? Explain.

Exercises 23 and 24 involve the following setting. Some women would like to have children but cannot do so for medical reasons. One option for these women is a procedure called in vitro fertilization (IVF), which involves injecting a fertilized egg into the woman's uterus.

23. **Prayer and pregnancy** Two hundred women who were about to undergo IVF served as subjects in an experiment. Each subject was randomly assigned to either a treatment group or a control group. Women in the treatment group were intentionally prayed for by several people (called *intercessors*) who did not know them, a process known as intercessory prayer. The praying continued for three weeks following IVF. The intercessors did not pray for the women in the control group. Here are the results: 44 of the 88 women in the treatment group got pregnant, compared to 21 out of 81 in the control group.[17]

Is the pregnancy rate significantly higher for women who received intercessory prayer? To find out, researchers perform a test of $H_0: p_1 = p_2$ versus $H_a: p_1 > p_2$, where p_1 and p_2 are the actual pregnancy rates for women like those in the study who do and don't receive intercessory prayer, respectively.

(a) Name the appropriate test and check that the conditions for carrying out this test are met.

(b) The appropriate test from part (a) yields a *P*-value of 0.0007. Interpret this *P*-value in context.

(c) What conclusion should researchers draw at the $\alpha = 0.05$ significance level? Explain.

(d) The women in the study did not know whether they were being prayed for. Explain why this is important.

24. **Acupuncture and pregnancy** A study reported in the medical journal *Fertility and Sterility* sought to determine whether the ancient Chinese art of acupuncture could help infertile women become pregnant.[18] One hundred sixty healthy women who planned to have IVF were recruited for the study. Half of the subjects (80) were randomly assigned to receive acupuncture 25 minutes before embryo transfer and again 25 minutes after the transfer. The remaining 80 women were assigned to a control group and instructed to lie still for 25 minutes after the embryo transfer. Results are shown in the table below.

	Acupuncture group	Control group
Pregnant	34	21
Not pregnant	46	59
Total	**80**	**80**

Is the pregnancy rate significantly higher for women who received acupuncture? To find out, researchers perform a test of $H_0: p_1 = p_2$ versus $H_a: p_1 > p_2$, where p_1 and p_2 are the actual pregnancy rates for women like those in the study who do and don't receive acupuncture, respectively.

(a) Name the appropriate test and check that the conditions for carrying out this test are met.

(b) The appropriate test from part (a) yields a *P*-value of 0.0152. Interpret this *P*-value in context.

(c) What conclusion should researchers draw at the $\alpha = 0.05$ significance level? Explain.

(d) The women in the study knew whether or not they received acupuncture. Explain why this is important.

Multiple choice: Select the best answer for Exercises 25 to 28.

Exercises 25 to 27 refer to the following setting. A sample survey interviews SRSs of 500 female college students and 550 male college students. Researchers want to determine whether there is a difference in the proportion of male and female college students who worked for pay last summer. In all, 410 of the females and 484 of the males say they worked for pay last summer.

25. Take p_M and p_F to be the proportions of all college males and females who worked last summer. The hypotheses to be tested are

(a) $H_0: p_M - p_F = 0$ versus $H_a: p_M - p_F \neq 0$.

(b) $H_0: p_M - p_F = 0$ versus $H_a: p_M - p_F > 0$.

(c) $H_0: p_M - p_F = 0$ versus $H_a: p_M - p_F < 0$.

(d) $H_0: p_M - p_F > 0$ versus $H_a: p_M - p_F = 0$.

(e) $H_0: p_M - p_F \neq 0$ versus $H_a: p_M - p_F = 0$.

26. The researchers report that the results were statistically significant at the 1% level. Which of the following is the most appropriate conclusion?

(a) Because the *P*-value is less than 1%, fail to reject H_0. There is not convincing evidence that the proportion of male college students in the study who worked for pay last summer is different from the proportion of female college students in the study who worked for pay last summer.

(b) Because the *P*-value is less than 1%, fail to reject H_0. There is not convincing evidence that the proportion of all male college students who worked for pay last summer is different from the proportion of all female college students who worked for pay last summer.

(c) Because the P-value is less than 1%, reject H_0. There is convincing evidence that the proportion of all male college students who worked for pay last summer is the same as the proportion of all female college students who worked for pay last summer.

(d) Because the P-value is less than 1%, reject H_0. There is convincing evidence that the proportion of all male college students in the study who worked for pay last summer is different from the proportion of all female college students in the study who worked for pay last summer.

(e) Because the P-value is less than 1%, reject H_0. There is convincing evidence that the proportion of all male college students who worked for pay last summer is different from the proportion of all female college students who worked for pay last summer.

27. Which of the following is the correct margin of error for a 99% confidence interval for the difference in the proportion of male and female college students who worked for pay last summer?

(a) $2.576\sqrt{\frac{0.851(0.149)}{550}+\frac{0.851(0.149)}{500}}$

(b) $2.576\sqrt{\frac{0.851(0.149)}{1050}}$

(c) $2.576\sqrt{\frac{0.880(0.120)}{550}+\frac{0.820(0.180)}{500}}$

(d) $1.960\sqrt{\frac{0.851(0.149)}{550}+\frac{0.851(0.149)}{500}}$

(e) $1.960\sqrt{\frac{0.880(0.120)}{550}+\frac{0.820(0.180)}{500}}$

28. In an experiment to learn whether Substance M can help restore memory, the brains of 20 rats were treated to damage their memories. First, the rats were trained to run a maze. After a day, 10 rats (determined at random) were given M and 7 of them succeeded in the maze. Only 2 of the 10 control rats were successful. The two-sample z test for "no difference" against "a significantly higher proportion of the M group succeeds"

(a) gives $z = 2.25$, $P < 0.02$.

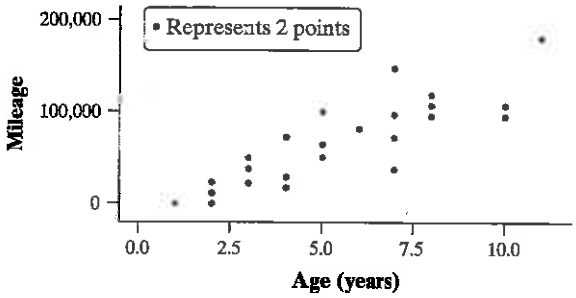
(b) gives $z = 2.60$, $P < 0.005$.

(c) gives $z = 2.25$, $P < 0.04$ but not < 0.02.

(d) should not be used because the Random condition is violated.

(e) should not be used because the Large Counts condition is violated.

Exercises 29 and 30 refer to the following setting. Thirty randomly selected seniors at Council High School were asked to report the age (in years) and mileage of their main vehicles. Here is a scatterplot of the data:



We used Minitab to perform a least-squares regression analysis for these data. Part of the computer output from this regression is shown below.

Predictor	Coef	Stdev	t-ratio	P
Constant	−13832	8773	−1.58	0.126
Age	14954	1546	9.67	0.000

s = 22723 R-sq = 77.0% R-sq(adj) = 76.1%

29. Drive my car (3.2)

(a) What is the equation of the least-squares regression line? Be sure to define any symbols you use.

(b) Interpret the slope of the least-squares line in the context of this problem.

(c) One student reported that her 10-year-old car had 110,000 miles on it. Find and interpret the residual for this data value. Show your work.

30. Drive my car (3.2, 4.3)

(a) Explain what the value of r^2 tells you about how well the least-squares line fits the data.

(b) The mean age of the students' cars in the sample was $\bar{x} = 8$ years. Find the mean mileage of the cars in the sample. Show your work.

(c) Interpret the value of s in the context of this setting.

(d) Would it be reasonable to use the least-squares line to predict a car's mileage from its age for a Council High School teacher? Justify your answer.