Chapter 6: Random Variables

1. Consider the following five sets of outcomes from random phenomena:

I. The total number of points scored in a randomly selected college football game.

II. Lifespan in hours of a randomly selected halogen light bulb.

III. The number of passengers in a randomly selected city bus.

IV. The airline of the next plane to land at O’Hare International Airport.

V. Length in inches of the next rattlesnake caught in a trap.

Which of the above are continuous random variables?

A. II and III only

AR. Incorrect. Recall that a continuous random variable must take values on an interval of real numbers. The precision of those values is limited only by how they are measured.

\*B. II and V only

BR. Correct. These variables take on real-numbered values. I, II take values on only a discrete set of numbers (the counting numbers), and IV takes on categorical values.

C. None of these are continuous random variables.

CR. Incorrect. Recall that a continuous random variable must take values on an interval of real numbers. The precision of those values is limited only by how they are measured.

2. Which of the following probability distributions of a discrete random variable *X* is a legitimate probability distribution?

A.

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | 1 | 2 | 3 |
| *p*(*x*) | 0.3 | 0.4 | 0.4 |

AR. Incorrect. The probabilities for all possible values must add up to 1.

B.

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | −1 | 0 | 1 |
| *p*(*x*) | 0.2 | 0.2 | 0.5 |

BR. Incorrect. The probabilities for all possible values must add up to 1.

\*C.

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | −1 | 0 | 1 |
| *p*(*x*) | 0.3 | 0.4 | 0.3 |

CR. Correct. It is possible for a random variable to take a negative value, and the sum of all the probabilities is 1.

3. In a particular game, a single card is randomly chosen from a box that contains 3 red cards, 1 green card, and 6 blue card. If a red card is selected, you win $2. If a green card is selected, you win $4. If a blue card is selected, you lose $1. Let *X* be the amount that you win. The expected value of *X* is

\*A. $0.40.

AR. Correct. The amount *X* takes the value $0 with probability 6/10, the value $2 with probability 3/10, and the value $4 with probability 1/10. The expected value (mean) of *X* is then given by μ*X* = Σ *xp*(*x*) = (-$1)(6/10) + ($2)(3/10) + ($4)(1/10) = –6/10 + 6/10 + 4/10 = $0.40.

B. $1.00

BR. Incorrect. You probably forgot to consider the chances of losing $1.

C. $1.60

CR. Incorrect. You may have miscalculated the impact of losing $1 when a blue card is drawn.

4. Let *Z* = the number students in Mr. Rooney’s English class who arrive late on a randomly selected day. The expected value of *Z* is 2. Which one of the following is the best interpretation of what this means?

A. We can be confident that at least 2 students will be late to Mr. Rooney’s class on a randomly selected day.

AR. Incorrect. The expected value of a random variable is also called the mean of the random variable. If at least two are late, the mean number who are late will likely be higher than two.

\*B. On average, the number of students who are late to Mr. Rooney’s class on a randomly selected day is 2.

BR. Correct. The expected value of a random variable is also called the mean of the random variable. Over many, many days, we expect the mean number of student who arrive late to be very close to 2.

C. There are 2 students in Mr. Rooney’s class who almost always arrive late.

CR. Incorrect. Expected value refers to the mean number of students who arrive late, but that number can vary from day to day. *Which* students are late can vary as well.

5. Let *X* = the number of times that a randomly selected customer visits a grocery store during a one-week period. Suppose that the probability distribution of *X* is as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *X* | 0 | 1 | 2 | 3 |
| *P*(X) | 0.1 | 0.4 | 0.4 | 0.1 |

Determine the probability that a randomly chosen customer visits the grocery store at least twice during a one-week period.

A. 0.9

AR. Incorrect. You found the probability of the event "a randomly chosen customer visits the grocery store at most twice during a one-week period.”

\*B. 0.5

BR. Correct. The event "a randomly chosen customer visits the grocery store at least twice during a one-week period" can be written in terms of *X* as . The probability of this event is *P*(2) + *P*(3) = 0.4 + 0.1 = 0.5 by the addition rule for disjoint events.

C. 0.4

CR. Incorrect. You found the probability of the event "a randomly chosen customer visits the grocery store exactly twice during a one-week period.”

6. Let *X* = the number of times that a randomly selected customer visits a grocery store during a one-week period. Suppose that the probability distribution of *X* is as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *X* | 0 | 1 | 2 | 3 |
| *P*(*X*) | 0.1 | 0.4 | 0.4 | 0.1 |

Which of the following calculations yield the standard deviation of *X*, σ*X*?

A. 

AR. Incorrect. Recall that standard deviation involves squared deviations from the mean.

B. 

BR. Incorrect. Recall that standard deviation represents the typical deviation of the variable from its expected value.

\*C. 

CR. Correct. The formula for standard deviation or a random variable is .

7. The time in minutes *X* that you must wait before a train arrives at your local subway station is a uniformly distributed random variable that takes on values between 5 minutes and 15 minutes. That is, the density curve of the distribution of *X* has constant height between *X* = 5 and *X* = 15 and height 0 outside this interval. Determine *P*(6 < *x* < 8).

A. 0.1

AR. Incorrect. You have determined the correct height of the density curve of *X*, but this is not the same as the desired probability.

B. 0.5

BR. Incorrect. Check to see that you have assigned the proper height to the density curve of *X*.

\*C. 0.2

CR. Correct. Since the total area under any density curve is 1, and the interval over which *X* is defined has length 10, the height of the curve must be 1/10 = 0.1. P(6 < *x* < 8) is the area under the density curve of *X* between 6 and 8, or the area of a rectangle with height 0.1 and width

8 – 6 = 2; that is, (0.1)(2) = 0.2.

8. If *X* and *Y* are random variables, and  which of the following is a condition for calculating  by using ?

A. *X* and *Y* are both normally distributed.

AR. Incorrect. The shape of the two distributions has no impact on whether or not .

\*B. *X* and *Y* are independent.

BR. Correct. The variance of the sum (or difference) of two random variables can be found by adding variances only if the two variables are independent.

C. *X* and *Y* are mutually exclusive.

CR. If *X* and *Y* are mutually exclusive, then they cannot both happen, which renders their sum meaningless.

9. The weight of a medium-sized orange selected at random from a large bin of oranges at a local supermarket is a random variable with mean μ = 12 ounces and standard deviation σ = 1.2 ounces. Suppose we independently select two oranges at random from the bin. The difference in the weights of the two oranges (the weight of the first orange minus the weight of the second orange) is a random variable with a standard deviation equal to

A. 0 ounces.

AR. Incorrect. Review the formula for the standard deviation of the difference of two independent random variables.

\*B. 1.70 ounces.

BR. Correct. Letting the independent random variables *X* and *Y* represent the weight of the first orange and the second orange, respectively, we have .

C. 2.88 ounces.

CR. Incorrect. You have found the variance of the difference!

10. The weight of a medium-sized orange selected at random from a large bin of oranges at a local supermarket is a Normally distributed random variable with mean μ = 12 ounces and standard deviation σ = 1.2 ounces. Suppose we independently select two oranges at random from the bin. What is the probability that the difference in the weights of the two oranges exceeds 3 ounces?

A. 0.0026

AR. Incorrect. Did you forget to standardize the weights of the oranges?

B. 0.0392

BR. Incorrect. This is the probability that the first orange’s weight exceeds the second one’s weight by 3 or more ounces. That is not quite what you were asked for! Read the question carefully!

\*C. 0.0784

CR. Correct. Letting the independent random variables *X* and *Y* represent the weight of the first orange and the second orange, respectively , we want the probability that  Because of symmetry this is equivalent to 

11. A widget manufacturer estimates that the total weekly cost in dollars, *C*, to produce *x* widgets is given by the linear function *C*(*x*) = 500 + 10*x*, where the intercept 500 represents the fixed cost of manufacturing any number of widgets and the slope 10 represents the variable cost of producing *x* widgets. Analysis of weekly widget production reveals that the number of widgets *X* produced in a week is a random variable with mean and standard deviation  What are the mean and the standard deviation of *C*?

\*A. 

 AR. Correct. The mean of a linear transformation is given by , and the standard deviation is given by , so  and .

B. 

BR. Incorrect. Recall that when you compute the standard deviation of a linear transformation the constant term *a* does not alter standard deviation.

C. 

CR. Incorrect. Recall that when you compute the mean and standard deviation of a linear transformation the constant term *a* affect the mean but does not alter standard deviation.

12. The daily total sales (excluding Saturday) at a small restaurant has a probability distribution that is approximately Normal with a mean of  and a standard deviation of  The probability the sales will exceed $700 on a given day is approximately

A. 0.9222.

AR. Incorrect. Recall that finding  for a Normal random variable *X* is equivalent to finding the area to the right of the value *x* under the appropriate Normal curve. You have found the area to the left of *x*.

B. 0.5778.

BR. Incorrect. Have you calculated the standard score for $700 correctly? Recall that .

\*C. 0.0778.

CR. Correct. 

13. A set of 10 playing cards consists of 5 red cards and 5 black cards. The cards are shuffled thoroughly, and we draw 4 cards one at a time and without replacement. Let *X* = the number of red cards drawn. The random variable *X* has which of the following probability distributions?

A. binomial distribution with parameters *n* = 10 and *p* = 0.5

AR. Incorrect. Since 4 cards were selected, the number of trials (draws) is 4, not 10. This could not possibly be a binomial distribution with parameter *n* = 10.

B. binomial distribution with parameters *n* = 4 and *p* = 0.5

BR. Incorrect. If the observations were independent, this would be the correct answer. However, the observations are *not* independent. If the first card is black, for example, the second is more likely to be red, because there are now more red cards than black cards remaining in the deck. In general, the probability of getting a particular color on a particular draw depends on the colors of the cards that were previously drawn.

\*C. neither (A) nor (B)

CR. Correct. This distribution is not binomial. The conditions for an experiment to be a binomial experiment are not satisfied because the four trials (draws) are not independent.

14. There are 20 multiple-choice questions on an exam, each having four possible responses, of which only one is correct. Each question is worth 5 points if answered correctly. Suppose that a student guesses the answer to each question, with her guesses from question to question being independent. If the student needs at least 40 points to pass the exam, the probability that she passes is closest to

A. 0.0609.

AR. Incorrect. The random variable *X* = “number correct out of 20” has a binomial distribution with *n* = 20 and *p* = 0.25. To get at least 40 points, the student must get at least 8 of the 20 questions correct. You have calculated the probability of the student getting exactly 8 questions correct.

\*B. 0.1018.

BR. Correct. The random variable X = “number correct out of 20” has a binomial distribution with *n* = 20 and *p* = 0.25. To get at least 40 points, the student must get at least 8 of the 20 questions correct. The desired probability, expressed in terms of *X*, is therefore *P*(*X* ≥ 8). Adding binomial probabilities for all values of *X* from 8 through 20 yields the desired answer.

C. 0.9591.

CR. Incorrect. The random variable *X* = “number correct out of 20” has a binomial distribution with *n* = 20 and *p* = 0.25. To get at least 40 points, the student must get at least 8 of the 20 questions correct. You have calculated the probability of the student getting at most 8 (that is, 8 or fewer) of the questions correct.

15. There are 20 multiple-choice questions on an exam, each having four possible responses, of which only one is correct. Each question is worth 5 points if answered correctly. Suppose that a student guesses the answer to each question, with her guesses from question to question being independent. The student’s expected (mean) score on this exam is

\*A. 25.

AR. Correct. The random variable *X* = “number correct out of 20” has a binomial distribution with *n* = 20 and *p* = 0.25. The mean of *X* is  The student’s score on the exam can be written as the linear function 5*X*, so the student’s mean score is 

B. 5.

BR. Incorrect. The random variable *X* = “number correct out of 20” has a binomial distribution with *n* = 20 and *p* = 0.25. You have calculated the mean of *X,* , but we want the mean score, student’s which is a linear function of *X*.

C. 50.

CR. Incorrect. This answer would be correct if guessing meant you had a 50% chance of being correct on each problem. Here, however, the chance of being correct is only 25%.

16. There are 20 multiple-choice questions on an exam, each having four possible responses, of which only one is correct. Each question is worth 5 points if answered correctly. Suppose that a student guesses the answer to each question, with her guesses from question to question being independent. The standard deviation of the student’s score on the exam is

\*A. 9.68.

AR. Correct. The random variable *X* = “number correct out of 20” has a binomial distribution with *n* = 20 and *p* = 0.25. The standard deviation of *X* is  The student’s score on the exam can be written as the linear function 5*X*, so the standard deviation of the student’s score is 

B. 1.94.

BR. Incorrect. The random variable *X* = “number correct out of 20” has a binomial distribution with *n* = 20 and *p* = 0.25. You have calculated the standard deviation of *X*, but we want the standard deviation of the student’s score, which is a linear function of *X*.

C. 93.75.

CR. Incorrect. The random variable *X* = “number correct out of 20” has a binomial distribution with *n* = 20 and *p* = 0.25. You have calculated the variance of the student’s score, σ25*X*, but we want the standard deviation, which is the square root of the variance.

17. In a certain large population, 70% are right-handed. You need a left-handed pitcher for your softball team and decide to find one by asking people chosen from the population at random. (We assume that once you do find a left-hander, he or she will be happy to join your team and will say yes.) Which of the following expression gives the probability that the first left-hander you finder is the fourth person you ask?

\*A. 

AR. Correct. We want *P*(*X* = 4), where *X* = “number of people you ask to get your first left-hander” is a geometric random variable with *p* = 0.3.

B. 

BR. Incorrect. Recall that *p* = the probability of a success and 1 – *p* = the probability of a failure in the geometric setting. What have you identified as a “success” and a “failure”?

C. 

CR. Incorrect. It appears you have misidentified this as a problem involving a binomial random variable!

18. In a certain large population, 70% are right-handed. You need a left-handed pitcher for your softball team and decide to find one by asking people chosen from the population at random. (We assume that once you do find a left-hander, he or she will be happy to join your team and will say yes.) Which of the following is closest to the probability that you will have to ask four or more people before finding your first left-hander?

A. 0.103

AR. Incorrect. This is the probability that you have to ask exactly four people.

B. 0.147

BR. Incorrect. This is the probability that you have to ask exactly three people.

\*C. 0.343

CR. Correct. The question is equivalent to asking for the probability that the first three people you ask are right-handed (and the first left-hander comes any time after that). Thus the answer is .

19. At a high school with 800 students, 80% of the students ride the school bus. If 20 students are selected randomly (without replacement) and we let *X* = the number of students in the sample who ride the bus, then *X* does not exactly have a binomial distribution. Why is it nevertheless appropriate to approximate probabilities for X using the binomial distribution for *n* = 20 and p = 0.8?

A. Since *np >* 10, we can still use the binomial distribution.

AR. Incorrect. This is one of the conditions for using the Normal distribution to approximate the binomial distribution. It is not a requirement for calculating binomial probabilities directly.

\*B. Because the sample is less than 10% of the population, it is appropriate to use the binomial distribution even though the samples are not strictly independent.

BR. Correct. This is the 10% rule for using the binomial distribution with random samples.

C. The binomial is always appropriate when sampling without replacement.

CR. Incorrect. Sampling without replacement means that the probability of a “success” on any given trial depends on the results of previous trials, since the composition of the population is changed (only slightly in this case) by each trial.

20. At a high school with 800 students, 80% of the students ride the school bus. If 20 students are selected randomly (without replacement) and we let *X* = the number of students in the sample who ride the bus, what is the probability that at least one of the students doesn’t ride the bus?

A. 0.0115

AR. Incorrect. This is the probability that none of the students rides the bus.

B. 0.0576

BR. Incorrect. This is the probability that exactly one student does not ride the bus.

\*C. 0.9885

CR. Correct. *X* has a binomial distribution with *n* = 20 and *p* = 0.8. The event “at least one student doesn’t ride the bus” is the complement of “All the students ride the bus.” So .