Chapter 10: Comparing Two Populations or Groups

1. Wilt and Bill, two basketball players, are having a free throw shooting contest. Wilt is known to make 75% of his free throws, and Bill is known to hit 85% of his free throws. Each of them takes 50 shots. Assuming that the shots are independent, what is the probability that Bill hits a higher percentage of his shots that Wilt?

A. 0.1038

AR. Incorrect. Think about whether your answer makes sense. If Bill is a better shooter, should the likelihood that he hits a higher percentage be greater than 0.5?

\*B. 0.8962

BR. Correct. The sampling distribution of means for the difference in proportion of shots hit by Bill and Wilt has a mean of 0.85 – 0.75 = 0.10 and a standard deviation of . So 

C. 0.9938

CR. It looks like you have miscalculated the standard deviation of hte difference of proportions. Check the formula and try again.

2. For a simple random sample of 100 cars of a certain popular model in 20012, it was found that 20 had a certain minor defect in the brakes. For an independent SRS of 400 cars of the same model in 2009, it was found that 50 had the same defect. Let *p*1 and *p*2 be the proportions of all cars of this model in 2012 and 2009, respectively, that have the defect. A 90% confidence interval for *p*1 – *p*2 is (approximately)

A. 0.075 ± 0.085.

AR. Incorrect. You computed a 95% confidence interval for *p*1 – *p*2 rather than the 90% confidence interval.

\*B. 0.075 ± 0.071.

BR. Correct.

The 90% confidence interval is given by

C. 0.075 ± 0.043.

CR. Incorrect. Did you include the appropriate critical *z\** for a 90% confidence interval?

3. Which of the following set of values for *n*1, *n*2, , and  satisfies the Normal/Large counts condition for constructing a confidence interval for the difference of two proportions, . (Assume that the 10% condition has been satisfied.)

A. *n*1 = 100, *n*2 = 100,  = 0.08,  = 0.10

AR. Incorrect. To use the interval, **, *n*1(1 − ), **, and *n*2(1 − ) must all be 10 or more. In this case, ** = (100)(0.08) = 8 is less than 10, so we cannot use the interval.

\*B. *n*1 = 100, *n*2 = 100, = 0.12, = 0.10.

BR. Correct. To use the interval, **, *n*1(1 − ), **, and *n*2(1 − ) must all be 10 or more. In this case, these quantities are, respectively, 12, 88, 10, and 90, all of which are at least 10, so the interval may be used.

C. *n*1 = 200, *n*2 = 200,  = 0.50,  = 0.04.

CR. Incorrect. To use the interval, **, *n*1(1 − ), **, and *n*2(1 − ) must all be 10 or more. In this case, **= (200)(0.04) = 8 is less than 10, so we cannot use the interval.

4. In a simple random sample of 100 households in 2000, 43 had some credit card debt. In another simple random sample of 150 households in 2012, 72 had some credit card debt. We want to construct a confidence interval for the difference in the proportion of households with credit card debt between 2012 and 2000. Which of the following is the correct standard error estimate for this interval?

\*A. 

AR. Correct. For a confidence interval for , the standard error is estimated by .

B. 

BR. Incorrect. It looks like you’ve used the pooled standard error formula, which is not appropriate for constructing a confidence interval. You also added fractions incorrectly!

C. .

CR. Incorrect you should not use the pooled standard error formula when constructing a confidence interval!

5. In a simple random sample of 100 households in 2000, 43 had some credit card debt. In another simple random sample of 150 households in 2012, 72 had some credit card debt. Which of the following is the correct test statistic for testing the hypothesis , that there is no difference in the proportion of households with credit card debt in these two years?

A. .

AR. Incorrect. You have not used the correct formula for the denominator of the test statistic! In addition, remember that when the null hypothesis is that there is no difference, you can use the pooled estimate for standard error.

B. 

BR. Incorrect. Remember that when the null hypothesis is that there is no difference, you can use the pooled estimate for standard error.

\*C. 

CR. Correct. When you are testing the null hypothesis that there is no difference between the two proportions, you can use the pooled estimate for standard error. Thus the test statistic is , where .

6. In a simple random sample of 100 cars of a certain popular model in 2010, it was found that 20 had a certain minor defect in the brakes. For an independent SRS of 400 cars of the same model in 2011, it was found that 50 had the same defect. Let *p*1 and *p*2 be the proportions of all cars of this model in 2010 and 2011, respectively, that have the defect. We wish to test  against . Which of the following is closest to the *P*-value for this test?

A. 0.0418.

AR. Incorrect. You calculated the test statistic without using the pooled sample proportion of successes, . Instead, you used the standard error for calculating a level C confidence interval for *p*1 – *p*2, which is based only on the individual sample proportions  and .

B. 0.0536.

BR. Incorrect. You found the *P*-value for a two-sided test. This test is one-sided!

\*C. 0.0268.

CR. Correct. The test statistic is  This produces a *P*-value of 0.0268 for a one-sided test.

7. A manufacturer receives parts from two suppliers. An SRS of 400 parts from Supplier 1 contains 20 defective parts, while an independent SRS of 100 parts from Supplier 2 contains 10 defective parts. Let *p*1 and *p*2 be the proportions of defective parts made by Supplier 1 and Supplier 2, respectively. Is there evidence of a significant difference in the proportions of defective parts made by these two suppliers? To answer this question, you test the hypotheses  against . Which of the following is closest to the *P*-value of the test?

A. 0.9398

AR. Incorrect. You have found the *P*-value for the one-sided alternative . Think about whether this answer makes any sense, given what the *P*-value measures!

\*B. 0.0602.

BR. Correct. The test statistic for a test of is  ,

where  is the pooled sample proportion of successes:  So  Since this test is two-sided, the *P*-value is 

C. 0.0301.

CR. Incorrect. You have found the *P*-value for the one-sided alternative .

8. SAT scores of entering freshmen at Enormous State University are approximately Normally distribution with a mean of and standard deviation , while the SAT scores of entering freshmen at the University of Modest Dimensions are approximately Normally distribution with mean and standard deviation . Independent random samples of 100 freshmen are selected from each university. Which of the following is closest to the probability that the sample mean from Enormous State University exceeds the sample mean from the University of Modest Dimensions?

\*A. 0.1446.

AR. Correct. The sampling distribution of the difference of means is approximately Normally distribution with mean  and standard deviation 

. So 

B. 0.0475.

BR. Incorrect. You evaluated , where the true population mean score for the University of Modest Dimensions. The distribution you are interested in is the difference of sample means, 

C. 0.8554.

CR. Incorrect. You evaluated *.*  You want to evaluate **.

9. Simple random samples are taken from two large populations, designated Population 1 and Population 2. Which of the following describes a situation in which the conditions for performing a two-sample *t*-test for the difference of two means for these populations have not been satisfied?

A.  The distribution of Sample 1 is symmetric and unimodal with no outliers, and the distribution of sample 2 is skewed left.

AR. Incorrect—the conditions have been satisfied. Sample 1 is small, but the distribution of the sample provides evidence that Population 1 is approximately Normally distributed. Sample 2 is large enough so that it is not necessary that Population 2 be Normal (because of the central limit theorem).

B.  The distributions of both samples are symmetric and unimodal with no outliers.

BR. Incorrect—the conditions have been satisfied. Both samples are small, but the distributions of the samples provides evidence that the populations are approximately Normally distributed.

\*C.  The distribution of Sample 1 is symmetric and unimodal with no outliers, and the distribution of Sample 2 is skewed left with no outliers.

CR. Correct. Since Sample 2 is small and it’s distribution is skewed, we do not have evidence that Population 2 is Normally distributed, so we have not met the Normal/Large Sample condition.

10. A sports physiologist wishes to compare the effects of two stepping heights (low and high) on heart rate in a step-aerobics workout. A sample of 50 adults in roughly similar physical condition was randomly divided into two groups of 25 subjects each. Group 1 did a standard step-aerobics workout using the low stepping height. The sample mean heart rate at the end of Group 1’s workout was beats per minute (bpm), with a sample standard deviation of bpm. Group 2 did the same workout but used the high stepping height. The sample mean heart rate at the end of Group 2’s workout was bpm, with a sample standard deviation of  bpm. Assume that conditions for inference have been met. Let  and represent the mean heart rates we would observe for the entire population of interest if all members of the population did the workout using the low and high stepping height, respectively. Suppose that the researcher wishes to test the hypotheses *versus* Which of the following is a correct expression for test statistic for this test?

\*A. 

AR. Correct. The test statistic for a test of the difference of means for two independent samples is .

B. .

BR. Incorrrect. The denominator—the standard error for the difference of two means—is not the sum of the standard deviation. Add variances!

C. 

CR. Incorrect. The variance for the difference of two means is the sum of the variances for each mean, not the difference.

11. A sportswriter wishes to see if a football filled with helium travels farther, on average, than a football filled with air. To test this hypothesis, the writer uses 18 male subjects, randomly divided into two groups of 9 subjects each. Group 1 kicks a football filled with helium to the recommended pressure, while Group 2 kicks a football filled with air to the same pressure. The sample mean yardage for Group 1 was  = 30 yards, with a sample standard deviation of *s*1 = 8 yards. The sample mean yardage from Group 2 was  = 26 yards, with a sample standard deviation of *s*2 = 6 yards. Let  represent the mean yardage observed for the entire population if all members of the population kicked a helium-filled football and an air-filled football, respectively. Assuming that conditions for a two-sample *t* procedure are have been satisfied and using the conservative value for the number of degrees of freedom, which of the following is a 90% confidence interval for ?

A. 4 ± 5.5 yards.

AR. Incorrect. You incorrectly used a standard Normal critical value rather than a *t* critical value when computing the margin of error.

\*B. 4 ± 6.2 yards.

BR. Correct. A level C confidence interval for in this situation would be

, where *t*\* = 1.860, the 90% critical value of the *t*(8) distribution. Thus, the 90% confidence interval is 

C. 4 ± 7.7 yards.

CR. Incorrect. You computed a 95% confidence interval rather than a 90% confidence interval.

12. Some agricultural researchers have conjectured that stem-pitting disease in peach-tree seedlings might be controlled through weed and soil treatments. An experiment was conducted to compare seedling growth with soil and weeds treated with one of two herbicides. In a field containing 10 seedlings, 5 were randomly selected and assigned to be treated with Herbicide A. The remaining 5 seedlings were treated with Herbicide B. Soil and weeds for each seedling were treated with the appropriate herbicide. At the end of the study period, the height (in centimeters) was recorded for each seedling. A 90% confidence interval for the difference  in mean seedling height for the two herbicides was found to be (0.2, 14.6). From this result, which of the following statements is correct?

A. The *P*-value for a test of  against would be greater than 0.10, since the interval doesn’t contain 0.

AR. Incorrect. The 90% confidence interval contains all differences for which we would fail to reject the null hypothesis at the  = 0.01 level. Because 0 is not in this interval confidence interval, we would reject  for α = 0.10. Thus, the *P*-value must be less than 0.10.

B. A 95% confidence interval would not include 0 either, since we would be even more confident that a significant difference exists between the two groups.

BR. Incorrect. As the confidence level increases, the interval gets wider, not narrower. So if the 90% confidence interval doesn’t include 0, then the 95% confidence interval may or may not include 0, depending on the data.

\*C. Neither (A) nor (B) is correct.

CR. Correct. See the responses to (A) and (B) for an explanation.

13. A drug company is testing a new medication for Attention Deficit Disorder (ADD). They have 40 eighth-grade volunteers who have been diagnosed with ADD. Which of the following experiments would call for matched pairs *t-*test?

A. Twenty randomly-selected students are treated with the new medication, twenty are treated with an established medication for ADD. After four weeks the students’ are tested for ADD symptoms.

AR. Incorrect. This is a completely randomized design that calls for comparison of two independent groups.

\*B. The students are divided into groups of two according to their academic achievement (the highest achievers are in one group, the next two in the second group, etc.). One randomly-selected member of each group is given the new medication and the other is treated with the established mediation. After four weeks, the students are tested for ADD symptoms.

BR. Correct. The groups of two are the “pairs,” randomly assigned to the two treatments. The appropriate null hypothesis is where = the true difference between ADD symptoms for pairs of students with similar academic achievements who are given the two different medications.

C. Neither of these experiments calls for a matched-pairs t-test.

CR. Incorrect. One of these experiments calls for matched pairs, (Hint: pairs are not necessarily perfect matches!).