Chapter 8: Estimating with Confidence

1. A random sample of 85 sixth-graders in a large city take a course designed to improve scores on a reading comprehension test. Based on this sample, a 90% confidence interval for the mean improvement in test scores for all sixth-graders in the city taking this course is found to be (12.6, 14.8). Which of the following are the sample mean and margin of error on which this interval is based?

\*A. Sample Mean = 13.7; Margin of Error = 1.1

AR. Correct. Recall that confidence intervals have the form *Point estimate ± margin of error*. So the sample mean (point estimate) is the midpoint of the interval (13.7) and the margin of error is half the width of the interval (1.1).

B. Sample mean = 13.7; Margin of Error = 2.2

BR. Incorrect. Recall that confidence intervals have the form: *Point estimate ± margin of error*.

C. Sample mean is unknown; Margin of Error = 2.2

CR. Incorrect. Recall that confidence intervals have the form: *Point estimate ± margin of error*. What is the point estimate in this case?

2. A random sample of 85 sixth-graders in a large city take a course designed to improve scores on a reading comprehension test. Based on this sample, a 90% confidence interval for the mean improvement μ in test scores for all sixth-graders in the city taking this course is found to be (12.6, 14.8). Which one of the following is a correct interpretation of this confidence interval?

A. Ninety percent of the 85 sixth-graders in the sample improved their reading comprehension scores by 12.6 to 14.8 points.

AR. Incorrect. The confidence interval estimates the mean score, not the scores of individuals in a sample.

B. The probability is 0.90 that the true mean improvement in reading comprehension is between 12.6 and 14.8.

BR. Incorrect. Once you have stated a confidence interval, the true mean score is either inside the interval or it isn’t. There is no probability associated with the true mean because it is a fixed value, not a random variable.

\*C. We are 90% confident that the true improvement in reading comprehension scores is captured by the interval 12.6 to 14.8.

CR. Correct. The interpretation of a confidence interval gives a range of plausible values for the true mean and a measure of the confidence we have in that range of values. (Interpreting the *interval* does not address what we mean by a specific confidence *level*.)

3. Agricultural researchers plant 100 plots with a new variety of corn and measure the mean yield for these plots in bushels per acre. They treat the 100 plots as a simple random sample of possible plots of corn (as researchers often do) and report a 95% confidence interval for the mean corn yield of (128.4, 131.6) bushels per acre. Which of the following is a correct interpretation of the 95% confidence level?

A. We are 95% confident that the true mean yield for this new variety of corn is captured by the interval (128.4, 131.6) bushels per acre.

AR. Incorrect. This is an interpretation of the confidence *interval*, not the confidence *level*. What does it mean to be 95% confident?

\*B. If many intervals were constructed this way from many independent sets of 100 plots, 95% of the intervals would capture the true mean corn yield.

BR. Correct. The confidence level describes “capture rate” for intervals constructed this way. The interval’s width was calculated so that 95% of the time the sample mean will be within one margin of error of the true mean.

C. If many intervals were constructed this way from many independent sets of 100 plots, 95% of the time the true mean corn yield would be in the interval (128.4, 131.6) bushels per acre.

CR. Incorrect. The true mean is a fixed parameter, so it is either in a specific interval or it is not. It’s the confidence interval that varies from sample to sample.

4. A random sample of size *n* is collected from a Normally distributed population with standard deviation σ. Using these data, a confidence interval is computed for the mean of the population. Which of the following actions would produce a new confidence interval with a smaller width (smaller margin of error), assuming that the same data were used?

A. Using a higher confidence level

AR. Incorrect. Increasing the confidence level will increase the margin of error. To increase the “capture rate” you must use a wider interval.

\*B. Using a lower confidence level

BR. Correct. The margin of error of a confidence interval is reduced by using a lower level of confidence. We can use a narrower interval if we are willing to reduce our “capture rate.”

C. Using a smaller sample size *n*

CR. Incorrect. Decreasing sample size increases the standard deviation of the statistic, thus creating a wider confidence interval.

5. A radio talk show host is interested in the proportion *p* of adults in his listening area who think that the drinking age should be lowered to 18. To find this proportion, he poses the following question to his listeners: “Do you think that the drinking age should be reduced to 18?” He asks listeners to phone in and vote “yes” or “no” depending upon their opinions. Of 200 people who phone in, 140 answer “yes.” The standard error of the sample proportion  of “yes” votes among those who phone in is

A. 0.21.

AR. Incorrect. Review the formula for the standard error of a proportion!

\*B. 0.032.

BR. Correct. , and the standard error of  is given by .

In this case, *n* = 200 and  = 140/200 = 0.7, so 

C. 0.00105.

CR. Incorrect. You forgot to take the square root when calculating the standard error of . Review the formula!

6. An inspector monitors large truckloads of potatoes to determine the proportion *p* of potatoes with major defects before the potatoes are used to make potato chips. She intends to compute a 95% confidence interval for *p*. To do so, she selects a simple random sample of 50 potatoes from a truckload of more than 2000 potatoes. Suppose that only 2 of the 50 potatoes sampled are found to have major defects. Which one of the following assumptions for inference about a proportion using a confidence interval is violated?

\*A. Large counts.

AR. Correct. In this case, *n* = 50 and  = 2/50, so ** = 2, which is less than 10. This assumption is therefore violated.

B. 10% condition.

BR. Incorrect. We are told that the inspector selected an SRS of 50 potatoes from the truckload of more than 2000 potatoes. The population size is more than 2000, which is more than 10 times as large as the sample size of 50. This condition is not violated.

C. There are no violations.

CR. Incorrect. An assumption isviolated here—review the conditions for a confidence interval for a proportion!

7. One hundred rats with mothers that were exposed to high levels of tobacco smoke during pregnancy were put through a simple maze. At the outset, the maze required the rats to make a choice between going left and going right. Eighty of the rats went right when running the maze for the first time. Assume that the 100 rats can be considered an SRS from the population of all rats born to mothers who were exposed to high levels of tobacco smoke during pregnancy. (Note that this assumption may or may not be reasonable, but researchers often assume that lab rats are representative of large populations, since they are often bred to have uniform characteristics.) Let *p* be the proportion of rats in this population that would go right when running the maze for the first time. A 90% confidence interval for *p* is

A. 0.8 ± 0.040.

AR. Incorrect. You neglected to include the critical value *z*\* for 90% confidence when calculating the margin of error of the confidence interval.

B. 0.8 ± 0.078.

BR. Incorrect. You computed a 95% confidence interval rather than the requested 90% confidence interval.

\*C. 0.8 ± 0.066.

CR. Correct. , and *z\** for 90% confidence is 1.645. So a 90% confidence interval is given by .

8. A noted psychic is tested for extrasensory perception. The psychic is presented, one at a time, with cards that are marked with one of four symbols: a star, a cross, a circle, or a square. Let *p* represent the probability that the psychic correctly identifies the symbol on the card in a random trial. Which of the following is the smallest number of trials you would have to conduct to estimate *p* to within ± 0.01 with 95% confidence? (Use the guess 0.25 as the value for *p*, since this would be the value of *p* if the psychic were merely guessing.)

A. 352

AR. Incorrect. You may have forgotten to include the critical value *z\**  in your calculation.

\*B. 7203

BR. Correct. Setting the formula for margin of error equal to 0.01, we get . Solving for *n*, we get . To be conservative, we always round up when calculating sample sizes, producing *n* = 1351.

C. 9604

CR. Incorrect. You used the conservative value *p*\* = 0.5 in your calculation instead of the suggested value *p*\* = 0.25. When you have reason to use a less conservative approach, do so!

9. To assess the accuracy of a kitchen scale, a standard weight, known to weigh 1.000 gram, is weighed a total of *n* times, and the mean  of the *n* weight measurements is computed. (If a 90% confidence interval for the mean calculated from these measurements does not contain 1.000, the scale is deemed inaccurate.) Suppose the scale readings are Normally distributed and that similar scales are known to measure such weights with a standard deviation of σ = 0.01 gram. Which of the following is the smallest value of *n* that will produce a 90% confidence interval with a margin of error of at most 0.001 grams?

A. 17

AR. Incorrect. Recall that a 90% confidence interval for a Normal population mean μ with standard deviation σ will have a margin of error of . Set  equal to 0.01 and solve for *n*. Be careful with the square root!

\*B. 271

BR. Correct. Recall that a 90% confidence interval for a Normal population mean μ with standard deviation σ will have a margin of error of . If we set  equal to 0.01 and solve for *n*, we get 268.96. (If we use 1.645 for *z\**, we get 270.6).

C. 385

CR. Incorrect. You used the incorrect standard Normal critical value *z*\* = 1.96 in your calculation. You would use *z*\* = 1.96 if you were estimating with 95% confidence rather than the requested 90%.

10. Which one of the following statements is true about Student’s *t* distribution?

A. The density curve of the *t* distribution is symmetric and centered at the true mean of the population.

AR. Incorrect. The *t­-*distribution always has a mean of 0.

\*B. The density curve of the *t* distribution has heavier “tails” than the density curve of the standard Normal distribution.

BR. Correct. This is a property of all *t* distributions. Extreme values are always more likely in a *t*-distribution that in a Normal distribution.

C. The density curve of the *t* distribution more closely resembles the density curve of the standard Normal distribution when the number of degrees of freedom is small.

CR. Incorrect. The *t* curve more closely resembles the standard Normal distribution when the number of degrees of freedom is *large*, not small.

11. The heights (in inches) of adult males on a small Pacific Island are believed to be Normally distributed with an unknown mean *μ*. The average height of a random sample of 25 adult males from the island is found to be  = 69.72 inches, with a sample standard deviation of *s* = 4.15 inches. A 99% confidence interval for *μ* is

\*A. 69.72 ± 2.32.

AR. Correct. A 99% confidence interval for the true mean height is given by. 2.792 is the critical *t\** for 99% confidence and 24 degrees of freedom. That is, the area between –2.797 and 2.797 in  is 0.99.

B. 69.72 ± 2.07.

BR. Incorrect. You used the incorrect critical value of the *t* distribution with 24 degrees of freedom when computing the interval.

C. 69.72 ± 11.61

CR. Incorrect. You used the sample standard deviation instead of the standard error of the mean.

12. You are thinking of using a *t* procedure to construct a 95% confidence interval for the unknown mean *μ* of a population for a random sample taken from the population. You suspect that the distribution of the population is not Normal and may in fact be moderately skewed. Which of the following statements is then correct?

A. You should not use the *t* procedure, since the population does not have a Normal distribution.

AR. Incorrect. This statement is too conservative. The *t* procedures are robust against non-Normality of the population if .

\*B. You can use the *t* procedure provided that .

BR. Correct. The *t* procedures are robust against non-Normality of the population if .

C. You can use the *t* procedure regardless of the sample size since *t* procedures are robust against non-Normality.

CR. Incorrect. The only time you would be able to use *t* procedures when the data are skewed is in the case of a large sample size (), and you cannot assume that such is the case here.